

### An example of system transformations and model reduction

```
A=[1 3;-1 -2];b=[1;0];c=[0 1];
```

```
poles=eig(A) =
-5.0000e-01 + 8.6603e-01i
-5.0000e-01 - 8.6603e-01i
% hence system stable
```

```
% Gramians
P=lyap(A,B*B') =
 2.5000e+00 -1.0000e+00
-1.0000e+00  5.0000e-01
Q=lyap(A',C'*C) =
 5.0000e-01  5.0000e-01
 5.0000e-01  1.0000e+00
```

```
% EVD of P, Q
[up,dp]=eig(P)
up =
-3.8268e-01 -9.2388e-01
-9.2388e-01  3.8268e-01
dp =
 8.5786e-02  0
 0  2.9142e+00
[uq,dq]=eig(Q)
uq =
-8.5065e-01  5.2573e-01
 5.2573e-01  8.5065e-01
dq =
 1.9098e-01  0
 0  1.3090e+00
```

```
% Associated matrices
phalf=up*sqrt(dp)*up' =
 1.5000e+00 -5.0000e-01
-5.0000e-01  5.0000e-01
qhalf=uq*sqrt(dq)*uq' =
 6.3246e-01  3.1623e-01
 3.1623e-01  9.4868e-01
qhp=qhalf*p*qhalf =
 6.5000e-01 -5.0000e-02
-5.0000e-02  1.0000e-01
[upq,dpq]=eig(qhp)
upq =
 9.9596e-01  8.9806e-02
-8.9806e-02  9.9596e-01
dpq =
```

```

6.5451e-01      0
0      9.5492e-02
sqrt(sqrt(dpq)) =
8.9945e-01      0
0      5.5589e-01

```

%%%%%%%% Balancing transformation %%%%

```

Tbal=inv(sqrt(sqrt(dpq)))*upq'*qhalf =
6.6874e-01  2.5544e-01
6.6874e-01  1.7508e+00

```

```

Pbal=Tbal*P*Tbal' =
8.0902e-01 -1.1102e-16
0      3.0902e-01

```

```

Qbal=inv(Tbal')*Q*inv(Tbal) =
8.0902e-01  2.7756e-17
0      3.0902e-01

```

```

Abal=Tbal*A*inv(Tbal) =
-2.7639e-01  8.9443e-01
-8.9443e-01 -7.2361e-01

```

```

Bbal=Tbal*B =
6.6874e-01
6.6874e-01

```

```

Cbal=C*inv(Tbal) =
-6.6874e-01  6.6874e-01

```

%%%%%%%% Degree-of-reachability transformation

```

Tp=up' =
-3.8268e-01 -9.2388e-01
-9.2388e-01  3.8268e-01

```

```

Pp=Tp*P*Tp' =
8.5786e-02  4.5103e-17
2.2204e-16  2.9142e+00

```

```

Qp=Tp*Q*Tp' =
1.2803e+00  1.7678e-01
1.7678e-01  2.1967e-01

```

```

Ap=Tp*A*inv(Tp) =
-8.5355e-01 -2.3223e-01
3.7678e+00 -1.4645e-01

```

```

Bp=Tp*B =
-3.8268e-01
-9.2388e-01

```

```

Cp=C*inv(Tp) =
-9.2388e-01  3.8268e-01

```

%%%%%%%% All-states-equally-reachable transformation

```
T2=inv(phalf) =
    1.0000e+00    1.0000e+00
    1.0000e+00    3.0000e+00
```

```
P2=T2*P*T2' =
    1.0000e+00   -2.2204e-16
   -3.3307e-16    1.0000e+00
```

```
Q2=inv(T2')*Q*inv(T2) =
    6.5000e-01   -1.2500e-01
   -1.2500e-01    1.2500e+00
```

```
A2=T2*A*inv(T2) =
   -5.0000e-01    5.0000e-01
   -1.5000e+00   -5.0000e-01
```

```
B2=T2*B =
    1.0000e+00
    1.0000e+00
```

```
C2=C*inv(T2) =
   -5.0000e-01    5.0000e-01
```

%%%%%%%% Degree-of-observability transformation

```
Tq=uq' =
   -8.5065e-01    5.2573e-01
    5.2573e-01    8.5065e-01
```

```
Pq=Tq*P*Tq' =
    2.8416e+00   -4.4721e-01
   -4.4721e-01    1.5836e-01
```

```
Qq=inv(Tq')*Q*inv(Tq) =
    1.9098e-01   -2.7756e-17
   -1.1102e-16    1.3090e+00
```

```
Aq=Tq*A*inv(Tq) =
   -7.2361e-01   -3.7889e+00
    2.1115e-01   -2.7639e-01
```

```
Bq=Tq*B =
   -8.5065e-01
    5.2573e-01
```

```
Cq=C*inv(Tq) =
    5.2573e-01    8.5065e-01
```

%%%%%%%%%% All-states-equally-observable transformation

```
T3=qhalf =
    6.3246e-01    3.1623e-01
    3.1623e-01    9.4868e-01
```

```
P3=T3*P*T3' =
    6.5000e-01   -5.0000e-02
   -5.0000e-02    1.0000e-01
```

```

Q3=inv(T3')*Q*inv(T3) =
    1.0000e+00    5.5511e-17
    1.1102e-16    1.0000e+00
A3=T3*A*inv(T3) =
   -2.0000e-01    1.4000e+00
   -6.0000e-01   -8.0000e-01
B3=T3*B =
    6.3246e-01
    3.1623e-01
C3=C*inv(T3) =
   -6.3246e-01    1.2649e+00

```

%%%%%%%% Summary of transformed and reduced systems

```

[Abal Bbal;Cbal 0] =
   -2.7639e-01    8.9443e-01    6.6874e-01
   -8.9443e-01   -7.2361e-01    6.6874e-01
   -6.6874e-01    6.6874e-01            0
ReducedSystem=[Ar Br;Cr 0] = % keep state x1; pole=-0.72
   -2.7639e-01    6.6874e-01
   -6.6874e-01            0

```

```

[Ap Bp;Cp 0] =
   -8.5355e-01   -2.3223e-01   -3.8268e-01
    3.7678e+00   -1.4645e-01   -9.2388e-01
   -9.2388e-01    3.8268e-01            0
ReducedSystem=[Ar Br;Cr 0] = % keep state x2; pole=-0.14
   -1.4645e-01   -9.2388e-01
    3.8268e-01            0

```

```

[A2 B2;C2 0] =
   -5.0000e-01    5.0000e-01    1.0000e+00
   -1.5000e+00   -5.0000e-01    1.0000e+00
   -5.0000e-01    5.0000e-01            0
ReducedSystem=[Ar Br;Cr 0] = % keep state x2; pole=-0.50
   -5.0000e-01    1.0000e+00
    5.0000e-01            0

```

```

[Aq Bq;Cq 0] =
   -7.2361e-01   -3.7889e+00   -8.5065e-01
    2.1115e-01   -2.7639e-01    5.2573e-01
    5.2573e-01    8.5065e-01            0
ReducedSystem=[Ar Br;Cr 0] = % keep state x2; pole=-0.27
   -2.7639e-01    5.2573e-01
    8.5065e-01            0

```

```

[A3 B3;C3 0] =
   -2.0000e-01    1.4000e+00    6.3246e-01
   -6.0000e-01   -8.0000e-01    3.1623e-01
   -6.3246e-01    1.2649e+00            0

```

```

ReducedSystem=[Ar Br;Cr 0] = % keep state x2; pole=-0.80
-8.0000e-01  3.1623e-01
1.2649e+00      0

```

```

%%%%%%%% Summary of transformed gramians

```

```

Sigma=Pbal=Qbal=sqrt(dpq) =
8.0902e-01      0
0      3.0902e-01
[Pp Qp]=[Tp*P*Tp' inv(Tp')*Q*inv(Tp)] =
8.5786e-02  4.5103e-17  1.2803e+00  1.7678e-01
2.2204e-16  2.9142e+00  1.7678e-01  2.1967e-01
[P2 Q2]=[T2*P*T2' inv(T2')*Q*inv(T2)] =
1.0000e+00 -2.2204e-16  6.2500e-01 -1.2500e-01
-3.3307e-16  1.0000e+00 -1.2500e-01  1.2500e-01
[Pq Qq]=[Tq*P*Tq' inv(Tq')*Q*inv(Tq)] =
2.8416e+00 -4.4721e-01  1.9098e-01 -2.7756e-17
-4.4721e-01  1.5836e-01 -1.1102e-16  1.3090e+00
[P3 Q3]=[T3*P*T3' inv(T3')*Q*inv(T3)] =
6.5000e-01 -5.0000e-02  1.0000e+00  5.5511e-17
-5.0000e-02  1.0000e-01  1.1102e-16  1.0000e+00

```

```

%%%%%%%%%%

```

```

% Modal transformation -> A diagonal -> T=inv(v)

```

```

[v,lam]=eig(A)
v =
8.6603e-01      8.6603e-01
-4.3301e-01 + 2.5000e-01i -4.3301e-01 - 2.5000e-01i
lam =
-5.0000e-01 + 8.6603e-01i      0
0      -5.0000e-01 - 8.6603e-01i

```

```

T=inv(v) =
5.7735e-01 - 1.0000e+00i      0 - 2.0000e+00i
5.7735e-01 + 1.0000e+00i      0 + 2.0000e+00i

```

```

Am=T*A*inv(T) = lam
-5.0000e-01 + 8.6603e-01i      0
0      -5.0000e-01 - 8.6603e-01i

```

```

Bm=T*B =
5.7735e-01 - 1.0000e+00i
5.7735e-01 + 1.0000e+00i

```

```

Cm=C*inv(T) =
-4.3301e-01 + 2.5000e-01i -4.3301e-01 - 2.5000e-01i

```

```

%%%%%%%%

```