Handout – ELEC 302

I/O and I/S/O representation of SISO linear systems	
I/O	**I/S/O**
variables: (u,y) $q\left(\frac{d}{dt}\right)y(t)=p\left(\frac{d}{dt}\right)u(t),\ n=\deg q\geq\deg p$ $u(t),y(t)\in\mathbb{R}$	variables: (u,x,y) $\frac{d}{dt}x(t)=Ax(t)+Bu(t),\ y(t)=Cx(t)+Du(t)$ $x(t)\in\mathbb{R}^n,\ \left(\begin{array}{c\|c} A & B \\ \hline C & D\end{array}\right)\in\mathbb{R}^{(n+1)\times(n+1)}$
Impulse response	
$q\left(\frac{d}{dt}\right)h(t)=p\left(\frac{d}{dt}\right)\delta(t)$ $H(s)=\mathcal{L}(h(t))=\frac{p(s)}{q(s)}$	$h(t)=D\delta(t)+Ce^{At}B,\ t\geq 0$ $H(s)=D+C(sI-A)^{-1}B$
Poles - characteristic roots - eigenfrequencies	
$\lambda_i,\ q(\lambda_i)=0,\ i=1,\cdots,n$	$\det(\lambda_i I-A)=0$
Zeros	
$H(z_i)=0\ \Leftrightarrow\ p(z_i)=0,\ i=1,\cdots,n$	$\det\begin{pmatrix} z_iI-A & -B \\ -C & -D\end{pmatrix}=0$
	Matrix exponential
	$e^{At}=\sum_{i=0}^{\infty}\frac{t^k}{k!}A^k\quad\Rightarrow\quad\frac{d}{dt}e^{At}=Ae^{At}=e^{At}A$ $\mathcal{L}(e^{At})=(sI-A)^{-1}$
BIBO stability	
$y=h*u$, requirement $\forall u\ni\\|u\\|_\infty<\infty\Rightarrow\\|y\\|_\infty<\infty$ $\Leftrightarrow\\|h\\|_1=\int_0^\infty\|h(t)\|dt<\infty$ $\Leftrightarrow\mathcal{R}e(\lambda_i)<0\Leftrightarrow$ poles $\in$ LHP	
Solution in the time domain	
$y(t)=y_{zi}(t)+y_{zs}(t)$ $y(t)=\sum_{i=1}^n c_ie^{\lambda_it}+\int_{0^-}^t h(t-\tau)u(\tau)d\tau$	$x(t)=x_{zi}(t)+x_{zs}(t)$ $x(t)=e^{At}x(0^-)+\int_{0^-}^t e^{A(t-\tau)}Bu(\tau)d\tau$ $y(t)=Ce^{At}x(0^-)+\int_{0^-}^t\underbrace{(D\delta(t-\tau)+Ce^{A(t-\tau)}B)}_{h(\cdot)}u(\tau)d\tau$ $y(t)=Ce^{At}x(0^-)+\int_{0^-}^t h(t-\tau)u(\tau)dt$
$\mathcal{L}$: Solution in the frequency domain	
$Y(s)=\frac{r(s)}{q(s)}+H(s)U(s)$	$X(s)=(sI-A)^{-1}x(0^-)+(sI-A)^{-1}BU(s)$ $Y(s)=C(sI-A)^{-1}x(0^-)+\underbrace{(D+C(sI-A)^{-1}B)}_{H(s)}U(s)$

Definition of state from I/O description. Let $H(s)=D+\frac{\bar{p}(s)}{q(s)}$, $\deg\bar{p}<\deg q$. Define w so that $q\left(\frac{d}{dt}\right)w(t)=u(t)$, $y(t)=\bar{p}\left(\frac{d}{dt}\right)w+Du(t)\Rightarrow x^T=\begin{pmatrix}w & w^{(1)} & \cdots & w^{(n-1)}\end{pmatrix}\in\mathbb{R}^n,\ n:$ degree of $q(s)$.

Various responses.
Zero-input or *free* response: response due to initial conditions alone.
Zero-state or *forced* response: response due to input (forcing function) alone (zero initial conditions).
Homogeneous solution: general form of free-response (arbitrary initial conditions).
Particular solution: forced response.
Steady-state response: response obtained for large values of time $T\to\infty$.
Transient response: full response minus steady state response.