Finite Difference System of Burgers' Equation

Consider again 1D unsteady Burgers'Equation:

$$\frac{\partial}{\partial t}y(x,t) = \nu \frac{\partial^2}{\partial x^2}y(x,t) - \frac{\partial}{\partial x}\left(\frac{y(x,t)^2}{2}\right) \qquad x \in [0,1], t \geq 0 \tag{1}$$

$$y(0,t) = y(1,t) = 0, t \geq 0 \;\text{ and }\; y(x,0) = y_0(x), x \in [0,1],$$

where $y(x,t)$ is the unknown function of time t and location $x \in \Omega \equiv [0,1]$; ν is a diffusion coefficient(viscosity parameter); and $y_0(x)$ is an initial condition. In this writeup, the initial condition is $y_0(x) = f(x) - f(0)$, where $f(x) = e^{-(15(x-0.5))^2)}$; $\nu = 0.1$; $t \in [0,1]$. Finite difference (FD) for spatial discretization is given by

$$\frac{d}{dt}\mathbf{y}(t) = \nu \mathbf{A}\mathbf{y}(t) + \mathbf{F}(\mathbf{y}), \tag{2}$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$ is the discrete Laplace operator; $\mathbf{F}(\mathbf{y}) = -\mathbf{y}. * \mathbf{A}_x\mathbf{y}$ with first-order discrete differential operator $\mathbf{A}_x \in \mathbb{R}^{n \times n}$ and '$.*$' denotes pointwise multiplication (note: $-\frac{\partial}{\partial x}\left(\frac{y(x,t)^2}{2}\right) = -y(x,t)\frac{\partial y(x,t)}{\partial x}$). Here the full-order dimension n is 100.