ELEC 302	HW #1	1/23/08

Due: 1/29/08 - 12 noon

Problem [1]. Given are the matrices listed below. Find their singular value decompositions (i.e. determine the singular values, right/left singular vectors) and compute the rank one approximations, which are optimal in the 2-norm. Use paper and pencil.

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 6 & 3 \\ -1 & 2 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

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Problem [2]. (a) Let $A \in \mathbb{R}^{n\times n}$ and $\det A \neq 0$. What is the relationship between the singular values of A and A^{-1}?

(b) Find the singular values of $A = \begin{pmatrix} p & -q \\ q & p \end{pmatrix}$. Explain your answer geometrically. Find $u \in \mathbb{R}^2$, $\|u\|_2 = 1$, such that $\|Au\|_2 = \sigma_1$. Explain.

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Problem [3]. Consider the following quantified representation of the letter "**X**":

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{8\times 7}$$

Determine the rank r of this matrix. Find an optimal approximant of rank $k = 1, \cdots, r$, and compute the error in each case. Compute the maximum of the absolute values of the entries of each error matrix and compare it with the theoretical value of the error. Finally, consider X and its approximants and error matrices as images and plot them using the following command sequence:

```
[n,m] = size(X);
figure(1)
imshow(X,64);
mag = 30;
truesize(1,[n*mag, m*mag]);
```

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Problem [4]. Approximation of the `MATLAB` images: text.png, tape.png, saturn.png. After starting `MATLAB`, type

```
text=imread('text.png');
tape=imread('tape.png');
saturn=imread('saturn.png');
text0=double(text)/256;
tape0=double(tape)/256; tape0=tape0(:,:,1);
saturn0=double(saturn)/256; saturn0=saturn0(:,:,1);
% Let Z be one of these images, for instance the Saturn image
Z=saturn0;
```

```
% In this case we need to scale the image
[mz,nz] = size(Z);
imshow(Z,64);
mag = 1/3;
truesize(1, [mz*mag, nz*mag]);
```

Compute the SVD of each iamge, using the command: `[U,S,V] = svd(Z);`

1. Plot the singular values on a logarithmic scale.
2. Compute approximants having error less than 10%, 5%, 2% of the largest singular value of Z. What is the rank of the corresponding approximants? also for each case compute the compression ratio (compression ratio is defined as the number of bytes required to store the approximant divided by the original image size in bytes.)
3. Now tile the image into four equal pieces. For each of the above errors, use SVD to approximate the sub-images and then reconstruct the complete image from them. Compute the compression ratio for this image. Compute the 2-norm error of the approximant and then compare the result with the previous one. Which method is better? which one requires more computations?
4. Attach with your homework the original image, the approximant *and* the error from the first and second methods, for the case the error is less than 2%.

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