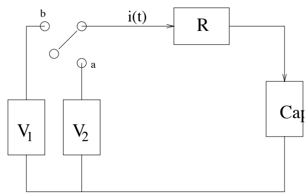
ELEC 302	HW #2	3/29/08

Due: 2/5/08 - 12 noon

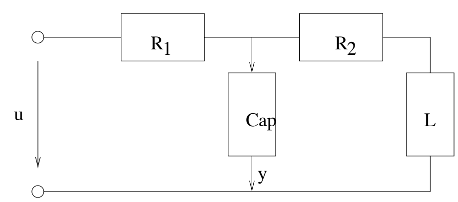
Problem [1]. The impulse response of an LTI system is $(e^{-t} - e^{-t}\cos t)\mathbb{I}(t)$, where $\mathbb{I}(t)$ denotes the unit step function. What is the step response? Find the input which gives rise to the output $(1 + e^{2t})\mathbb{I}(t)$, for the same LTI system. ■

Problem [2]. Consider the circuit shown in the figure with $R = 1\Omega$ and Cap $= 1F$. The switch is supposed to have been at position a for a long time (say, since $t = -\infty$). At $t = 0$, it goes to position b. Find the current $i(t)$, $t \geq 0$, for the following values of the two batteries:



(i) $V_1 = 0V$, $V_2 = 1V$; (ii) $V_1 = 1V$, $V_2 = 0V$; (iii) $V_1 = 1V$, $V_2 = 1V$.
Using your answers for (i), (ii), (iii), argue that the current $i(t)$ can be considered as a sum of the circuit's zero-state response and zero-input response. ■

Problem [3]. Consider the RLC circuit shown in the figure. The notation is as follows: u is the input voltage, y the output current, x_1 is the voltage across the capacitor and x_2 the current through the inductor. **(a)** Write



state and output equations and determine n, m, p and A, B, C, D. Write the i/o equation of this system in the form $q(\frac{d}{dt})y = p(\frac{d}{dt})u$ (determine $q(s)$ and $p(s)$). **(b)** Assume zero initial conditions and unit value for all elements. Compute the matrix exponential e^{At}, the state x, the transfer function and the impulse response. Hence compute the step response. Identify the transient and steady state parts of each one of these functions. **(c)** From the transfer function of this system, or otherwise, deduce the differential equation relating y and u. ■

Problem [4]. Put the following systems of differential equations in the form $\dot{x} = Ax + Bu$:

$$\begin{aligned}\dot{x}_1 &= -2x_1 + x_2 + x_3,\\ \dot{x}_2 &= \quad x_1 + x_2 - 2x_3,\\ \dot{x}_3 &= \quad x_1 + x_2 - 2x_3.\end{aligned}$$

Compute e^{At} by using the definition of the matrix exponetial (i.e. computing the powers of A). What is $x(t)$ if $x_1(0) = x_2(0) = 0$ and $x_3(0) = 1$? Investigate the stability of this system. Finally, does there exist an initial condition $x(0)$ such that $x(t)$ remains bounded (i.e. does not go to infinity) as $t \to \infty$? If so, find all such initial conditions and explain their relationship with the eigenvectors of A. ■