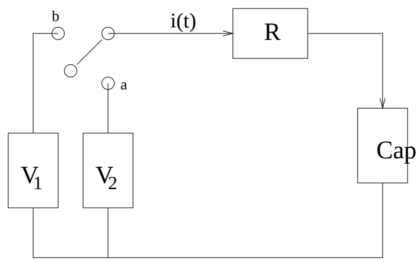
ELEC 302	HW #2

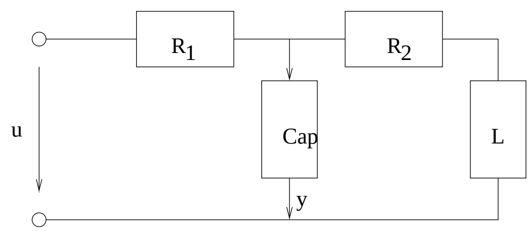
Due: see Organizational Info

Problem [1]. Consider the circuit shown in the figure with $R = 1\Omega$ and Cap $= 1F$. The switch is supposed to have been at position a for a long time (say, since $t = -\infty$). At $t = 0$, it goes to position b. Find the current $i(t)$, $t \geq 0$, for the following values of the two batteries:



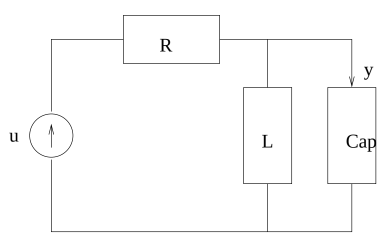
(i) $V_1 = 0V$, $V_2 = 1V$; (ii) $V_1 = 1V$, $V_2 = 0V$; (iii) $V_1 = 1V$, $V_2 = 1V$.
Using your answers for (i), (ii), (iii), argue that the current $i(t)$ can be considered as a sum of the circuit's zero-state response and zero-input response. ■

Problem [2]. Consider the RLC circuit shown in the figure. The notation is as follows: u is the input voltage, y the output current, x_1 is the voltage across the capacitor and x_2 the current through the inductor. **(a)** Write state and output equations and determine n, m, p and A, B, C, D. Write the i/o



equation of this system in the form $q(\frac{d}{dt})y = p(\frac{d}{dt})u$ (determine $q(s)$ and $p(s)$). **(b)** Assume zero initial conditions and unit value for all elements. Compute the state x, the transfer function and the impulse response. Hence compute the step response. ■

Problem [3]. Given is the RLC circuit shown below.



The input is the current u provided to the circuit, while the output is the current y through the capacitor Cap.

(a) Write these equations in the form $\dot{x} = \hat{A}x + \hat{B}u$, $y = \hat{C}x + \hat{D}u$, where x_1 is the current through the inductor and x_2 is the voltage across the capacitor; what are the matrices $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$?

(b) For Cap $= 1F$, $L = 1H$ and $R = 1\Omega$, compute the matrix exponential $e^{\hat{A}t}$. Hence for $u = \mathbb{I}$, compute the states x_1, x_2, and the output y. Identify the transient and steady state parts of each one of these functions.

(c) Compute the transfer function of this system; from this, or otherwise, deduce the differential equation relating y and u, and compute the impulse response. ■