ELEC 302	HW #4	2/12/08

Due: 2/19/08 - 12 noon

Problem [1]. Consider the system given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find the transfer function of this system and hence the differential equations describing the system in terms of inputs and outputs (i.e. no states).

(b) Determine controllability from u_1, u_2, and u. If any of these systems is not controllable, find a basis for the controllable space. ■

Problem [2]. Consider the inverted penduli system given in the *Case Studies* for the following values of the parameters: $M = 2$, $m = 1$, $\ell_1 = 1$, $\ell_2 = 1$, $g = 10$.
(a) Is the system stable? (b) Is the system controllable? Find the dimension and a basis for the controllable space. Is it possible to transfer each one of the initial states

$$\tilde{x} = [1 \;\; 0 \;\; 0 \;\; 0]^T, \;\; \hat{x} = [1 \;\; 1 \;\; 0 \;\; 0]^T, \;\; \bar{x} = [0 \;\; 0 \;\; 1 \;\; 1]^T$$

to the zero state? Furthermore, is it possible to transfer the state of the system from the initial state $\hat{x}$ to the final state $\bar{x}$? Justify your answers. ■

Problem [3]. Consider the RLC system given in the *Case Studies* with the following values of the parameters: $C_1 = C_2 = 1$, $R = R_1 = R_2 = 1$, $L_1 = L_2 = 1$. Consider also the states:

$$\hat{x} = [0 \;\; 1 \;\; 0 \;\; 0]^T, \; \tilde{x} = [0 \;\; 0 \;\; 0 \;\; 1]^T$$

(a) Is the system stable? Is the system controllable? Find the dimension and a basis for the controllable space.
(b) Explain whether there exists an input $\hat{u}$ which will transfer the state of the system from 0 to $\hat{x}$. Is it possible to transfer the state of the system from $\hat{x}$ to $\tilde{x}$? Justify your answer. ■