ELEC 302	HW #4

Due: see Organizational Info

Problem [1]. Consider the dynamical system described by $\dot{x} = Ax + Bu$, $y = Cx$, where

$$A = \begin{bmatrix} 0 & 1 & 3 \\ -1 & -2 & -2 \\ -1 & -1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad C = [a \;\; b \;\; c].$$

Compute the eigenvalue decomposition of $A = V\Lambda V^{-1}$; is A diagonalizable? Explain. Hence compute the matrix exponential e^{At}. Finally compute the real constants a, b, c such that the impulse response of this system contains a single pure exponential (i.e. no exponentials times t are allowed). How many different solutions are there and how are they related to eigenvectors of A? ■

Problem [2]. *Aircraft AFTI-16 lateral motion.* The state equations are given by $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} -0.746 & 0.006 & -0.999 & 0.0369 \\ -12.9 & -0.746 & 0.387 & 0 \\ 4.31 & 0.024 & -0.174 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0012 & 0.0092 \\ 6.05 & 0.952 \\ -0.416 & -1.76 \\ 0 & 0 \end{bmatrix}$$

The state is $x = [\beta \;\; \dot{\phi} \;\; r \;\; \phi]^*$, where β is the slip-angle, $\dot{\phi}$ is the roll rate, r is the yaw rate and ϕ is the roll angle. The control inputs are $u = [u_1 \;\; u_2]^*$, where u_1 is the aileron deflection and u_2 is the rudder deflection.

Find the eigenvalues of A and hence determine stability. Moreover, find all initial conditions which will *not* excite the oscillatory modes of the aircraft. Compute the *steady-state* values of the states for $u_1 = \mathbb{I}$, $u_2 = 0$ and $u_1 = 0$, $u_2 = \mathbb{I}$ (unit step). Find the impulse response h from the rudder deflection u_2 to the roll rate, i.e. $y = \dot{\phi}$. Identify the transient and state state parts of h. ■

Problem [3]. Consider the RLC system given in the *Case Studies* with the following values of the parameters: $C_1 = C_2 = 1$, $R = R_1 = R_2 = 1$, $L_1 = L_2 = 1$.

(a) Given the states as shown in the handout, write state and output equations.

(b) Is the system stable?

(c) Compute and sketch the impulse response $h(t)$. Hence compute the transfer function $H(s)$ of this system.

(d) From $H(s)$ compute the differential equation relating the input u_1 and the output x_2. Using the procedure discussed in class, construct a state for this differential equation, determine the associated matrices A, B, C and compute the transfer function. Is the answer what you expect? ■

Problem [4]. *Extra Credit* 20%. In problem **[3]** above, let the input be u_1 and the output be x_2. Let also the state space system, with states as shown in the handout, be given in terms of the matrices (A_o, B_o, C_o), while the state and output matrices obtained from $H(s)$ using the *construction-of-state procedure* described in class, are denoted by (A_n, B_n, C_n) (where the subscripts 'o' and 'n' refer to 'old' and 'new' states, respectively). Define the matrices

$$R(A_o, B_o) = [B_o, \; A_oB_o, \; A_o^2B_o, \; A_o^3B_o], \; R(A_n, B_n) = [B_n, \; A_nB_n, \; A_n^2B_n, \; A_n^3B_n] \in \mathbb{R}^{4\times 4},$$

and define $T = R(A_n, B_n)[R(A_o, B_o)]^{-1} \in \mathbb{R}^{4\times 4}$. Show that the relations

$$A_n = TA_oT^{-1}, \quad B_n = TB_o, \quad C_n = C_oT^{-1},$$

hold. Therefore the new and old states are related by means of the transformation T, i.e. $x_n = Tx_o$. For this particular example, write down explicitly how the old state variables x_1, x_2, x_3, x_4 relate to the new ones. ■