ELEC 302	**HW #5**	**3/12/08**

Due: 3/19/08 - 12 noon

Problem [1]. Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Consider the three systems resulting when the input is u_1, u_2 and $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, respectively. Compute the infinite controllability grammians of these three systems (if they exist), and hence deduce the energy required to reach the i^{th} unit vetor e_i using u_1, u_2 and u. Which states are easiest/most difficult to reach in each case? ■

Problem [2]. Controllability of the hot-air balloon. Let $w = 0$, i.e. consider only one input (the heat added to the air in the balloon); the values of the parameters are as given in the Case Studies handout.

Find the minimal energy input which will transfer the state $[0 \;\; 1 \;\; 1]^*$ to $[0 \;\; 0 \;\; 2]^*$ (i.e. double the height and bring the speed to zero) in one and ten units of time, respectively. Compute the inputs, their maximum amplitudes, and their energy in the two cases; plot these inputs. ■

Problem [3]. *Aircraft AFTI-16 lateral motion.* The state equations are given by $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} -0.746 & 0.006 & -0.999 & 0.0369 \\ -12.9 & -0.746 & 0.387 & 0 \\ 4.31 & 0.024 & -0.174 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0.0012 & 0.0092 \\ 6.05 & 0.952 \\ -0.416 & -1.76 \\ 0 & 0 \end{bmatrix}$$

The state is $x = [\beta \;\; \dot{\phi} \;\; r \;\; \phi]^*$, where β is the slip-angle, $\dot{\phi}$ is the roll rate, r is the yaw rate and ϕ is the roll angle. The control inputs are $u = [u_1 \;\; u_2]^*$, where u_1 is the aileron deflection and u_2 is the rudder deflection.

Find the eigenvalues of A and hence determine stability. Moreover, find all initial conditions which will *not* excite the oscillatory modes of the aircraft. Compute the *steady-state* values of the states for $u_1 = \mathbb{I}$, $u_2 = 0$ and $u_1 = 0$, $u_2 = \mathbb{I}$ (unit step). Find the impulse response h from the rudder deflection u_2 to the roll rate, i.e. $y = \dot{\phi}$. Identify the states which are most difficult/easy to reach in $T = 1$ and $T = 10$ units of time. ■