ELEC 302	HW #6

Due: see Organizational Info

Problem [1]. As in problem **[1]** of the previous assignment, consider the RLC system with the following values of the parameters: $C_1 = C_2 = 1$, $R = R_1 = R_2 = 1$, $L_1 = L_2 = 1$. Consider also the states:

$$\hat{x} = [1 \;\; 0 \;\; 0 \;\; 0]^T, \; \tilde{x} = [0 \;\; 0 \;\; 0 \;\; 1]^T$$

(a) Compute the infinite reachability gramian. (b) Hence find the state which is the easiest and most difficult to reach. (c) Find an input which will steer the system from 0 to $\hat{x}$ in an infinite amount of time. Repeat for $\tilde{x}$. What is the required energy in each case? Plot the two inputs on the same graph. ■

Problem [2]. (Problem **[2], HW #4**.) *Aircraft AFTI-16 lateral motion.* The state equations are given by $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} -0.746 & 0.006 & -0.999 & 0.0369 \\ -12.9 & -0.746 & 0.387 & 0 \\ 4.31 & 0.024 & -0.174 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0.0012 & 0.0092 \\ 6.05 & 0.952 \\ -0.416 & -1.76 \\ 0 & 0 \end{bmatrix}$$

The state is $x = [\beta \;\; \dot{\phi} \;\; r \;\; \phi]^*$, where β is the slip-angle, $\dot{\phi}$ is the roll rate, r is the yaw rate and ϕ is the roll angle. The control inputs are $u = [u_1 \;\; u_2]^*$, where u_1 is the aileron deflection and u_2 is the rudder deflection.

Given a reachable state x, the minimal energy required to reach it in T units of time is $x^* P^{-1}(T) x$. Compute the infinite reachability grammians P_1, P_2, and P for inputs: u_1, u_2, u, respectively. Show that $P = P_1 + P_2$; therefore verify that the eigenvalues of P_i are less than those of P, $i = 1, 2$ (this can be expressed compactly by means of the inequality $P \geq P_i \Rightarrow P - P_i$, $i = 1, 2$, is positive semi-definite). Hence determine the energy required to reach each one of the states $x = e_i$, $i = 1, 2, 3, 4$, where e_i is the ith unit vector. Comment on the relative difficulty of reaching each one of these states. ■

<u>*Suggestion*</u>: the use of `Matlab` for the preceding problems is recommended. Please attach the m-files used and the corresponding diary.

Problem [3]. (*Extra Credit 25%*) Given the system $\dot{x} = Ax + Bu$, let $\mathcal{P}(T)$ be the corresponding grammian at time T, $\hat{x}$ a reachable state, and $\hat{\xi}$ be such that $\hat{x} = \mathcal{P}(T)\hat{\xi}$. Show that among all inputs u which steer the state of the system from 0 at time 0, to $\hat{x}$ at time T, the input $\hat{u}(t) = B^* e^{A^*(T-t)}\hat{\xi}$, $0 \leq t \leq T$, has the smallest energy, i.e.

$$< u, u > \;\; \geq \;\; < \hat{u}, \hat{u} > \quad \text{where} \quad < f, g > := \int_0^T f^*(t) g(t) dt.$$

Hint: show that $< \hat{u}, u - \hat{u} > = 0$. ■