ELEC 302	Test #2	4/11/07

2 hours; closed book; no computers/calculators; two informative sheets allowed; show all work. All questions (a)–(h) have equal weight; answer any 7. `Sign the pledge.`

Problem [1]. Consider the system defined by $A = \begin{pmatrix} -1 & 0 \\ 0 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $C = [1 \;\; 2]$.

(a) Compute the infinite controllability and the infinite observability gramians and hence determine the states which are the most difficult/easiest to reach/observe.

(b) The following states are given: $x_1 = [0 \;\; 0]^T$, $x_2 = [1 \;\; 0]^T$, and $x_3 = [0 \;\; 1]^T$. Consider the minimal energy inputs which will steer the system from (i) x_1 to x_2, (ii) x_2 to x_1, (iii) x_3 to x_2. The initial time is zero, and you are aloted an infinite amount of time to perform these transfers ($T = \infty$). What is the energy of each of the three inputs achieving the above transfers?

(c) Find (i) the Hankel singular values of this system, (ii) a balancing transformation, and (iii) the reduced system of order one obtained by balanced truncation of the state which is least observable and least controllable. ■

Problem [2]. Let

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(d) Find the controllability gramian and the minimal energy input u_1 which will transfer the state of the system from zero to x_1 in T units of time.

(e) Find the minimal energy input u_2 which will transfer the state of the system from x_1 to zero in T units of time. What is the relationship between u_1 and u_2 and how do these functions behave as T tends to zero?

(f) Let $C = [1 \;\; 0]$. *Statement*: the state space matrices $\hat{A} = A^T$, $\hat{B} = C^T$, $\hat{C} = B^T$ can be obtained from A, B, C by change of basis.

Show that the statement holds, by computing a transformation T such that $\hat{A} = TAT^{-1}$, $\hat{B} = TB$, $\hat{C} = CT^{-1}$. ■

Problem [3]. Given is the system defined by:

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 1 & \alpha \\ 1 & 3 \\ 0 & 1 \end{pmatrix}, \; C = \begin{pmatrix} 1 & \beta & 0 \end{pmatrix}, \; \alpha, \; \beta \in \mathbb{R}.$$

(g) First determine the number of inputs, outputs and states of this system. Then, find a basis for the (i) controllable space of the system as a function of α, and a basis for its (ii) unobservable space as a function of β. Thus determine the extent to which the state of the system can be recovered from measurements of the output? Explain. (Both α and β are real parameters.) ■

Problem [4]. Given are the matices $K = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$ and $L = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

(h) First, show that they are positive definite. Then find a transformation that will simultaneously diagonalize them by congruence. Hence compute (i) the generalized eigenvalues of the pair (K, L), and (ii) the eigenvectors of KL^{-1} as a function of the eigenvectors of $L^{-1}K$. ■