# Communication Power Optimization in a Sensor Network with a Path-Constrained Mobile Observer

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We present a procedure for communication power optimization in a network of randomly distributed sensors with an observer (data collector) moving on a fixed path. The key challenge in using a mobile observer is that it remains within communication range of any sensor for a brief duration, and inability to transfer data in this duration leads to data loss. We establish that the process of data collection can be modeled by a queue with deadlines, where arrivals correspond to the observer entering the range of a sensor and a missed deadline means data loss. The queuing model is then used to identify the combination of system parameters that ensures adequate data collection with minimum power. The results obtained from the queuing analogy take a simple form in the asymptotic regime of dense sensor networks. Additionally, for sensor networks that cannot tolerate data loss, we derive a tight bound on minimum sensor separation that ensures that no data will be lost on account of mobility. We present two examples to illustrate our results, from which it is seen that power reduction by two orders of magnitude or more is typical relative to a static sensor network. The scenarios chosen for power comparisons also provide guidelines on the choice of path, if such a choice is available.

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## 1. INTRODUCTION

It has now been known for several years that mobile data collection [Grossglauser and Tse 2001; Diggavi et al. 2002] can provide better throughput scaling, or equivalently increase network lifetime, as compared to a static wireless sensor network (WSN) [Gupta and Kumar 2000; Chakrabarti et al. 2004; El Gamal 2005]. The fundamental advantage of mobility is that the observer

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can opportunistically communicate with a sensor when the two are close. The power savings that accrue from short-range communication, however, come at the cost of uncertainty in data collection. Moreover, several combinations of system parameters can ensure collection of the same amount of data, while leading to very different power consumptions. To address this issue, we propose a power optimization technique for data collection in a mobile observer WSN. We demonstrate that the optimum combination of system parameters leads to power savings of several orders of magnitude. Our mobility model incorporates defining features of the paths of public transport vehicles such as buses, trains, and elevators, which makes it more realistic than random mobility models.

The novel contributions of this article are as follows. First, we develop a queuing model to analyze the process of data collection by a mobile observer in a network with finitely many nodes. The arrival of a task at the queue is equivalent to a sensor entering the communication range of the observer, the queuing delay is related to the number of sensors within range at a given time, and a missed deadline means data loss. We derive expressions of the arrival process and the distribution of deadlines for the queue, and also prove that sensor arrivals can be closely approximated by a Poisson process. Using the queuing model, we quantify the success in data collection as a function of the communication range and data rate. Since the range and data rate also determine communication power, our model can be used as an optimization tool prior to sensor deployment to discover the set of power-minimizing parameter values for a specified maximum data loss fraction. We demonstrate our technique with a detailed example in Section 4.3.

In a mobile observer WSN, data is lost when the observer passes through a region of high local node density, so that not all sensors succeed in sending data to the observer while within range. This loss is inevitable if sensors are independently distributed; however, some WSN applications cannot tolerate data loss. Our second contribution extends the advantages of mobile data collection to these applications. We discover a constraint on minimum sensor separation that guarantees zero data loss with a mobile observer. The condition we derive is sufficient for zero data loss, and although it is not necessary, we show that it is tight.

Our third contribution is to show that, as node density increases, the results obtained from queuing analysis take a particularly simple form and lend themselves to analytical characterization without the need for queuing based numerical analysis. Our conclusion from the asymptotic analysis is that, in a dense network, almost all sensors that enter within range are able to transfer data successfully, provided that the data communication rate equals or exceeds the data collection rate by the desired fraction of sensors. As a consequence, if we require that the data loss fraction should not exceed $F_{\max}$, then the range need only be large enough for a fraction $(1 - F_{\max})$ of sensors to enter within range.

To demonstrate the utility of our results, we present two examples. In the first example, we calculate that with correct choice of system parameters the network power is typically reduced by two orders of magnitude or more relative to a static WSN. The second example deals with a disk-shaped WSN with the observer moving on a concentric circular path. This case throws light on several practical issues, such as edge effects, and the consequence of a model mismatch between the actual mobility pattern and our mobility model. We derive the optimal path radius for minimizing network power. This example also serves as a design guideline in situations where there are multiple paths over which the observer may move, and the problem is to choose the one that minimizes network power.

Most of the related work on mobile WSNs can be classified into one of two categories. The first class of work is inspired by Gupta and Kumar [2000] and studies large network scaling laws, giving insights on how capacity [Grossglauser and Tse 2001; Diggavi et al. 2002], power, or delay [Gamal et al. 2004] scales as the number of nodes increases. The dramatic improvement in capacity scaling due to mobility observed in Grossglauser and Tse [2001] and Diggavi et al. [2002] is an especially noteworthy contribution in this class. In contrast, the second class of research focuses on network architecture, and the actual design of protocols [Ailawadhi 2002; Shah et al. 2003; Kansal et al. 2004; Tong et al. 2003], the performance of which is mostly studied via simulations. Our approach, originally proposed in Chakrabarti et al. [2003], is distinct from both, in that our results are applicable to networks with finitely many nodes, yet based on rigorous analysis. In essence, we solve a power optimization problem pertinent to mobile WSNs. Power saving with mobility has also been explored in Rao and Kesidis [2004], Gandham et al. [2003], and Wu et al. [2004], but in contrast to our research, these treat the mobility pattern itself to be variable.

The remainder of this article is organized as follows. In Section 2, we describe our WSN model. We formulate our problem, and pose the important questions to be answered in Section 3. Section 4 studies the tradeoff between power and data loss in our network. Conditions for zero data loss are derived in Section 4.4. Section 5 is devoted to the asymptotic data collection analysis for dense networks. A power comparison between static and mobile observer WSNs is presented in Section 6. Section 7 summarizes our conclusions and discusses future directions. Appendix A describes the condition for guaranteeing zero data loss, and Appendix B gives the proof for Theorem 5.1.

## 2. SYSTEM MODEL

Here, we first provide a general description of a WSN, and the roles played by sensors and the observer. This is followed by a description of the observer mobility pattern. Last, we outline the model for single-hop communication between static sensors and the mobile observer.

### 2.1 Model of a WSN

The WSN consists of $N$ static sensor nodes that are independently and uniformly distributed over a planar surface with area $A$ (see Figure 1). Sensors collect information and send it to a common mobile observer $O$. The network is homogeneous, that is, all sensors are identical. Thus each node has the same amount of energy (battery), uses the same communication range, and the same data rate $D$ to transmit. Nodes do not perform power control. The above

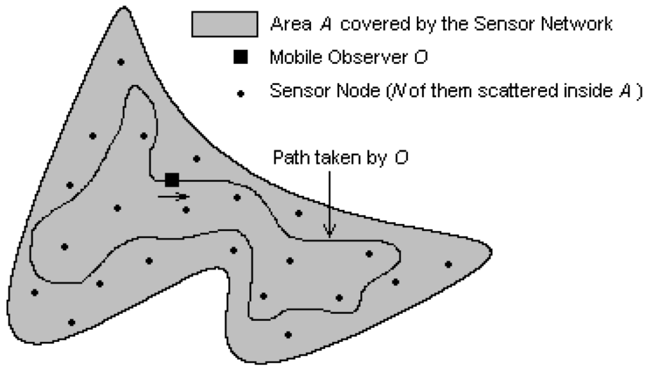


Fig. 1. Diagram of a WSN with a mobile observer.

assumptions are usually true for networks with cheap unsophisticated sensors that can be rapidly deployed.

Each sensor collects *nonredundant* information at a fixed rate $D_{sens} = \frac{D_{net}}{N}$, where $D_{net}$ is the rate at which the entire network collects information.[1] The above assumption is motivated by a bit-conservation principle—data of the same quality can be obtained by a dense network of low-resolution sensors as by a sparse network of high-resolution sensors [Ishwar et al. 2003]. Note that data correlation among nodes is automatically accounted for by the above assumption. If we did not account for correlation, then $D_{sens}$ would be independent of $N$, and $D_{net}$ would grow linearly with $N$. Alternatively, the nodes may collect redundant information at a fixed rate and employ Slepian-Wolf coding [Slepian and Wolf 1973] to eliminate redundancy. In either case, specifics of sensing and distributed source-coding are beyond the scope of this article. We will focus on how information which is free from redundancy is communicated.

Since sensors collect as well as communicate data at identical rates, it is concluded that the time $T$ taken to communicate information collected by a sensor in one cycle (defined in Section 2.2) is also fixed. Finally, sensors are energy constrained but the observer is not, since the observer derives its energy from a mobile vehicle.

### 2.2 Observer Mobility Model

The observer $O$ repeatedly traverses a deterministic path inside $A$. This path is fixed and cannot be chosen at will. However, we do assume that the path has the following properties:

—The set of points on the path is topologically path-connected, meaning that every point on the path is reachable from every other point by moving along the path.

—The path can be approximated by a straight line over distances of the order of the communication range of a sensor. In other words, the radius of curvature at each point on the path is large compared to the communication range.

[1]In practice, $D_{net}$ increases with $A$, often linearly. Since $A$ is fixed for us, our analysis is not influenced by the nature of this dependence.

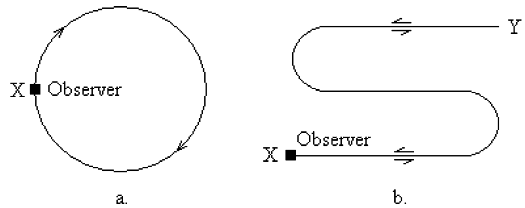


Fig. 2. Motion of the observer on a (a) Closed path and (b) a path that is not closed.

—As the observer moves on its path, it covers a *strip* of width $2R$ within which sensors can communicate with it.[2] This strip contains all points that are within distance $R$ from at least one point on the path of the observer. We require the path of the observer to be such that this strip covers all points in the network, but does not overlap with itself. As a consequence, the total area covered by this strip equals the area of the network.

Vehicular paths often consist of long stretches of low-curvature curves, and rarely traverse the same area multiple times. Furthermore, a path is suitable for data collection only from points in its vicinity. These were motivating ideas behind the proposed model. The model is simplified to ensure tractability. We realize that it is not suitable for all paths; however, it provides a good approximation for a large set of paths. We also believe that our model is more realistic than random mobility models, because the paths of transport vehicles have a well-defined sense of direction, and almost never resemble random curves.

Paths may either be *open* (Figure 2(b)). or *closed* (Figure 2(a)). When the observer, starting from some point and moving on its closed path returns to its starting point, we say that it has completed a *cycle*. If the path is open, then the *cycle* begins at one end of it path and ends at the other. In Figure 2(a), the observer returns to its starting point $X$ at the end of each cycle, whereas, in (Figure 2(b)), the observer completes one cycle from $X$ to $Y$, and, in the next cycle, the observer retraces its path from $Y$ to $X$. The speed of the observer is fixed at $v$ and the time taken to complete one cycle is denoted as $T_{cycle}$.

The following relation can be derived from the path properties:

$$A = 2RvT_{cycle}. \tag{1}$$

The equality

$$TD = T_{cycle}D_{sens} \tag{2}$$

captures the fact that all the information collected by a sensor at a rate $D_{sens}$ in a duration $T_{cycle}$ is transmitted at a rate $D$ in a duration $T$ to the observer.

### 2.3 Communication Model

We assume single-user single-hop communication over an AWGN channel with bandwidth and noise power spectral density normalized to unity. The channel attenuation exponent is $\gamma$. We also assume that communication takes place over

[2] $R$ is related to the communication range. A precise definition is given in Section 2.3.

continuous time slots of length $T$ that are long enough for the asymptotic capacity theorem for Gaussian channels to be invoked. All channels are assumed to be time-invariant. Thus, from a physical layer perspective, our channel model is

$$Y = \frac{X}{(d)^{\gamma/2}} + N, \tag{3}$$

where $Y$ is the channel output, $X$ is the input, $d$ is the distance between transmitter and receiver, and $N$ is additive white Gaussian noise.

For successful data transfer, the sensor and the observer must communicate continuously over a duration $T$. In our formulation, this condition is both necessary and sufficient for data transfer. The above requirement imposes constraints on the communication range. Let $\mathbf{S} = \{S_1, S_2, \ldots, S_N\}$ denote the set of sensors as well as their positions. For the observer to be within range of each sensor in the network at some point on its path, the range should be at least

$$R = \sup_i \left( \inf_t \|S_i - O(t)\|_2 \right), \tag{4}$$

where $O(t)$ is the time-varying position of the observer, and $\|.\|_2$ is the Euclidean norm. In other words, if we were to draw a circle of radius $R$ at every point on the path of the observer, then the union of the areas of all these circles would be just sufficient to contain all the sensors. Additionally, the observer must *remain* within range for at least a duration $T$. Taking this into account, the communication range for sensors must be chosen to satisfy[3]

$$R_m \geq R' = \sqrt{R^2 + (vT/2)^2}. \tag{5}$$

Choosing the communication range to be $R'$ ensures that a sensor will remain in range of the observer long enough to transfer its data. Therefore, (5) is a necessary condition for *all* sensors in the network to be successful in transferring data to the observer. Conversely, (5) is not necessary if data loss can be tolerated.

Interestingly, (5) is not a sufficient condition for successful data collection from all sensors. Due to the independent sensor distribution, at times there may be too many sensors in range contending to send data to the observer. If the rate of communication is not sufficiently high, there may not be enough time for all sensors to send their data to the observer, leading to data loss.

Our characterization of data loss is motivated by packet-based communication, where a data packet is either received or dropped. No credit is given for partially received data. Sensors are assumed to have knowledge (acquired through a suitable protocol [Chakrabarti et al. 2003]) that enables them to predict whether the observer will remain in range for the entire data transfer. Therefore, we will assume that all communications are successful, and data loss is due to sensors that never transmitted because of predictable communication failure.

The above assumptions permit the development of a powerful tool that can be incrementally advanced to include practical effects such as communication

[3]The straight-line approximation for the path is implicit here.

errors, inaccurate knowledge of node deadlines, and irregular vehicle speeds. However, a treatment of these and other issues is beyond the scope of this article.

## 3. PROBLEM FORMULATION

The average[4] communication power per sensor $P(R_m, D)$ and the average fraction of nodes that fail to send information to the observer $F(R_m, D)$ are both functions of the communication range $R_m$ and the data rate $D$. The average network power $P_{net}$ is a summation of the average node powers, and can be expressed in terms of the functions $P$ and $F$ as

$$P_{net} = (1 - F)NP. \tag{6}$$

We are faced with the task of designing the communication system for a WSN, that is, of choosing the parameters $R_m$ and $D$ optimally. The area of the network, the path over which the observer is constrained to move, and the number of sensors are parameters that are specified. The optimization problem can be stated as

$$\min_{F(R_m,D)\leq F_{\max}} P_{net}. \tag{7}$$

The following are questions that must be answered to meet the above goal, and to evaluate the effectiveness of a mobile observer relative to a static one:

(1) How can we characterize the tradeoff between the data loss fraction and the communication power? The problem is to find the combination of range and data rate that achieves minimum power for a specified data loss fraction.
(2) Certain WSN applications may not be able to afford data loss. In such cases, how can we deterministically guarantee zero data loss?
(3) How does the power consumption of mobile observer WSNs compare with that of static observer WSNs?

The rest of the article will address the above questions.

## 4. THE POWER-DATA LOSS TRADEOFF

In this section, we derive the functions for power $P(R_m, D)$ and loss fraction $F(R_m, D)$. The dependence of $P$ on $R_m$ and $D$ is well known. To describe $F$, however, we will first need to establish the analogy between the data collection process and a queuing system, and use this queuing formulation to get our answers. The procedure for determining $F$ will be explained with an example.

The function $P$ is derived as follows. In a cycle of duration $T_{cycle}$, a sensor collects $T_{cycle}D_{sens}$ bits, which are transferred to the observer at a data rate $D$ in a duration $T_{cycle}D_{sens}/D$. Therefore, a sensor transmits only for a fraction $D_{sens}/D$ of the cycle duration. Hence, $P_t$, the power during actual transmission,

[4]The average is with respect to different outcomes of the process of random scattering of sensors. Averaging is meaningful in a scenario where node locations are a priori unknown.

is related to $P$, the average transmission power over one cycle as

$$P_t = \frac{PD}{D_{sens}}. \quad (8)$$

Here, $D$ is the capacity of the AWGN channel over which sensors communicate with the observer, given by

$$D = \log_2\left(1 + \frac{P_t}{R_m^{\gamma}}\right), \quad (9)$$

from which we derive that

$$P = \frac{D_{sens} R_m^{\gamma}(2^D - 1)}{D}. \quad (10)$$

$D$ is actually a lower bound on capacity, since $R_m$ is the *maximum* separation between the observer and a communicating sensor. Equivalently, $P$ is an upper bound on communication power. However, the assumption that all nodes consume the same power $P$ ensures fairness, and is suitable in a setup where cheap sensors do not have the ability to perform power control. Moreover, the lifetime of a network is governed by sensors that consume the most power and die early.

Note that $P$ increases monotonically with both $R_m$ and $D$. It is also intuitive that $F$ is monotonically nonincreasing with $R_m$ and $D$ since longer deadlines and a faster service rate can only reduce the number of missed jobs. As a consequence, $P_{net}$ is also monotonically increasing in $R_m$ and $D$.

The dependence of $F$ on $R_m$ and $D$ is difficult to model for the following reasons. First, the fraction of unsuccessful nodes $F$ depends on the actual node locations, which are an outcome of a process of random scattering. Averaging over all possible outcomes is challenging. Second, the data-loss fraction depends on the sequence in which the observer communicates with sensors. It seems natural that the value of $F$ should correspond to the sequence that minimizes data loss. However, finding this optimal sequence in our framework is not straightforward.[5] Third, it is difficult to quantify the influence of $R_m$. The choice of $R_m$ influences $F$ in two different ways. When the chosen $R_m$ satisfies (5), data loss is only due to insufficiently high data rate and contention among sensors in areas of high local node density. On the other hand, if $R_m$ does not satisfy (5), then there is another component of the data loss in the form of sensors that never enter within range of the observer with the potential to transfer data. To discover how $F$ varies with $R_m$ and $D$, we will establish that the process of data collection can be mathematically modeled as a single-server queue with deadlines.

Before moving on to the queuing analogy, we introduce another parameter $f_b$, which is the fraction of time the observer is busy communicating. The relation

[5]We conjecture that this scheduling problem is NP-hard. Interestingly, earliest deadline first scheduling, which appears to be a promising candidate in our scenario, is provably suboptimal [Bhattacharya and Ephremides 1989].

between $f_b$ and $F$ is

$$f_b = \frac{(1-F)NT}{T_{cycle}}. \tag{11}$$

It follows from (2), (6), (10), and (11) that the average network power can be written in terms of $f_b$ as

$$P_{net} = f_b R_m^{\gamma}(2^D - 1). \tag{12}$$

Equations (11) and (12) will be useful for analyzing the power-data loss tradeoff in a dense network.

### 4.1 Data Collection as a Queue

As the observer moves, new sensors come in range, while ones that were in range go out of range. Since sensors are independently distributed, there may be imbalances in local node density leading to contention among sensors in regions of high density. If the data rate is insufficient, then there is not enough time for all sensors to communicate with the observer while in range, leading to data loss. Therefore, the process of data collection from a field of independently distributed sensors bears resemblance to a single-server queue with deadlines. Here, we establish the queuing analogy mathematically.

The event that a new node comes in range is termed an *arrival*. The observer may be busy when a new arrival occurs, in which case the node must wait in order to send its data. This corresponds to *queuing* of arrivals. When the observer finishes communicating with a node, it immediately starts communicating with another (if there is another node waiting for its turn). If there are multiple arrivals waiting in the queue, the observer chooses one based on some rule, which we call the *queue discipline*. In our framework, it is not useful to start communicating with a sensor node that will not stay within range long enough to transfer all its data, since partial data transfer is treated as failure. For each sensor there is a maximum waiting time or *laxity*, which is a function of its distance from the path of the observer. If the observer does not start communicating with the node before this time, it will be impossible for the sensor to send all its data. This is depicted in Figure 3. We assume that the observer has accurate knowledge of node deadlines, as if guided by an oracle. In Chakrabarti et al. [2003], the authors outlined a communication protocol explaining how knowledge of node deadlines can be acquired in practice.

To describe the queue, we need to specify its *arrival process*, the *distribution of deadlines*, the *service mechanism* which provides a statistical description of service times, and the *queue discipline* used to schedule arrivals. We start by introducing our notation. Let $t_0$ be the time when the observer starts moving. As the observer moves, sensors enter within range of communication. By convention, we will call the $i$th sensor to enter within range $S_i$. Let $t_i$ be the time when $S_i$ enters within range. The interarrival times are denoted $T_i = t_i - t_{i-1}$. $S_i$ remains within range of the observer for $d_i$ seconds, by which time communication must be completed to avoid data loss. We will refer to $d_i$ as the *deadline* of $S_i$. The time taken by $S_i$ to communicate its information to the observer is denoted by $c_i$, which is analogous to the *service time* of the queue. In our setup,

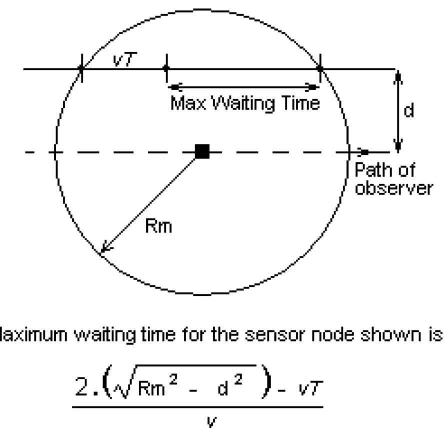


Fig. 3. Relation between waiting time and data loss: the sensor is at a distance d from the path of the observer. If the sensor enters within range at t = 0, then the observer must start communicating with the node before t = Maximum waiting time (laxity), failing which the node will go out of range before it can transfer all its data.

$c_i = T$ for all sensors. Another quantity of interest to us is the *laxity* $l_i = d_i - c_i$, which is the time by which communication must have *begun*.[6]

The characteristics of our queue are described as follows.

4.1.1 *Arrival Process.* In a time interval of length $t$, the observer moves a distance $vt$. If $R_m$ satisfies (5), then nodes within an area $2Rvt$ enter within range with the potential to transfer data.[7] The arrival process of the queue is derived in the following lemma.

LEMMA 4.1. *If the communication range $R_m$ satisfies* (5)*, then sensors enter within range of the observer with the following interarrival PDF:*

$$p_{T1}(t) = \frac{2RvN(A - 2Rvt)^{N-1}}{A^N}, \qquad 0 \leq t \leq \frac{A}{2Rv}. \tag{13}$$

PROOF. In a time duration $t$, the observer travels a distance $vt$. Nodes within an area $2Rvt$ that were previously out of range now come within range. Let $p_{T1}(t)$ be the PDF of interarrival times. Then[8]

$$\begin{aligned} \int_0^t p_{T1}(x)dx &= \text{probability that at least one node arrives in time } t && (14)\\ &= 1 - (\text{probability that no node arrives in time } t) && (15)\\ &= 1 - \left(\frac{A - 2Rvt}{A}\right)^N. && (16) \end{aligned}$$

[6]The terms *laxity* and *deadline* have been borrowed from the processor scheduling literature.
[7]If (5) is satisfied with strict inequality, then some nodes may be encountered more than once in a cycle. In that case, the observer will only acknowledge the arrival with the longest deadline.
[8]This follows from the fact that nodes are independently and uniformly distributed and the probability for any single node to be outside the area $2Rvt$ equals $(A - 2Rvt)/A$. Note that the time $t$ cannot exceed $(A/2Rv)$ because $(A/2Rv) = T_{cycle}$ from (1).

Taking the derivative on both sides with respect to $t$ yields (13), which is the PDF of interarrival times. □

Corollary 4.2. *If the communication range $R_m$ does not satisfy* (5)*, then sensors enter within range of the observer with the following interarrival PDF:*

$$p_{T2}(t) = \frac{2vN\sqrt{R_m^2 - (\frac{vT}{2})^2}\left(A - 2vt\sqrt{R_m^2 - (\frac{vT}{2})^2}\right)^{N-1}}{A^N}, \qquad 0 \le t \le \frac{A}{2Rv}. \quad (17)$$

Proof. Same as in Lemma 4.1, with $R$ replaced by $\sqrt{R_m^2 - (vT/2)^2}$. □

Obtaining the arrival process is necessary for describing the queue. However, the PDF of interarrival times given by Lemma 4.1 is rarely encountered in queuing theory, which limits its usefulness. Interestingly, the PDF of interarrival times can be approximated by an exponential PDF. Such an approximation is motivated by the observation that the PDF of interarrival times is memoryless for small $t$, and it is known that the exponential PDF is the only continuous PDF with the memoryless property. We approximate the interarrival PDF $p_{T1}(t)$ with

$$p^*_{T1}(t) = \frac{2RvN}{A}\exp\left(-\frac{2RvN}{A}t\right), \qquad t \ge 0, \quad (18)$$

and we approximate $p_{T2}(t)$ with

$$p^*_{T2}(t) = \frac{2vN\sqrt{R_m^2 - \left(\frac{vT}{2}\right)^2}}{A}\exp\left(-\frac{2vN\sqrt{R_m^2 - \left(\frac{vT}{2}\right)^2}}{A}t\right), \qquad t \ge 0. \quad (19)$$

As a consequence of (18) and (19), the arrival process can be treated as a Poisson process.

4.1.2 *Distribution of Deadlines.* The distribution of deadlines $d_i$ is derived in the following lemma.

Lemma 4.3. *If $R_m$ satisfies* (5)*, the PDF of service deadlines $d_i$ is*

$$p_{D1}(d) = \frac{dv^2}{4R\sqrt{R_m^2 - (vd/2)^2}}, \qquad \frac{2\sqrt{R_m^2 - R^2}}{v} \le d \le \frac{2R_m}{v}. \quad (20)$$

Proof. The PDF of deadlines may be derived as follows. The deadline for a sensor node at a distance $r$ from the path of the observer is

$$d(r) = \frac{2\sqrt{R_m^2 - r^2}}{v}. \quad (21)$$

Note that $r$ is uniformly distributed from 0 to $R$ as a consequence of the independent and uniform spatial distribution of sensors, and the assumption that the path can be approximated by a straight line over short distances. Hence, the PDF of waiting times may be obtained by transforming this uniform PDF using (21). □

COROLLARY 4.4. *If $R_m$ satisfies* (5)*, the PDF of sensor laxity $l_i$ is*

$$p_{L1}(d) = \frac{(d+T)v^2}{4R\sqrt{R_m^2 - (v(d+T)/2)^2}}, \qquad \frac{2\sqrt{R_m^2 - R^2}}{v} - T \le d \le \frac{2R_m}{v} - T. \tag{22}$$

PROOF. $l_i = d_i - T$. □

COROLLARY 4.5. *If $R_m$ does not satisfy* (5)*, the PDF of service deadlines $d_i$ is*

$$p_{D2}(d) = \frac{dv^2}{4\sqrt{R_m^2 - (vT/2)^2}\sqrt{R_m^2 - (vd/2)^2}}, \qquad T \le d \le \frac{2R_m}{v}, \tag{23}$$

*and the PDF of sensor laxity $l_i$ is*

$$p_{L2}(d) = \frac{(d+T)v^2}{4\sqrt{R_m^2 - (vT/2)^2}\sqrt{R_m^2 - (v(d+T)/2)^2}}, \qquad 0 \le d \le \frac{2R_m}{v} - T. \tag{24}$$

PROOF. The proof is as earlier with $R$ replaced by $\sqrt{R_m^2 - (vT/2)^2}$. □

4.1.3 *Service Mechanism.* The distribution of service times depends on the distribution of data among sensors. Since sensors collect equal amounts of data, service times are also equal.

We consider only *nonpreemptive* and *nonidling* service mechanisms in the context of our queue. In a nonpreemptive service mechanism, a task that has begun must reach completion before another task can start. In situations where task-switching has little or no associated cost, preemption can sometimes improve performance. In a wireless network, preemption would carry a protocol overhead, making it undesirable. An idling policy allows the server to remain idle in anticipation of arrivals with short deadlines even when the queue is not empty. We consider only nonidling policies in this article because, as we will see later, our queuing system will be working under overloaded conditions, where idling will carry no advantage.

4.1.4 *Queue Discipline.* Unlike the arrival process and the distribution of deadlines, the queue discipline is not fixed by the network parameters. The observer can freely choose the rule by which it will serve arrivals, and must make the best choice to minimize data loss. In our framework, the observer traces the same path repeatedly in a field of static sensors. Consequently, after the first cycle, the sequence of arrivals is known. Therefore, the observer can act as a clairvoyant scheduler and find the schedule that minimizes data loss.

A clarification may be necessary at this point. The determinism described above should not be misinterpreted to mean that the queuing formulation is unnecessary. At the design stage, node locations are not known. Sensors must be designed to meet performance criteria in the absence of such knowledge. The queuing formulation acts as a useful tool that predicts network behavior on average without knowledge of node locations.

In summary, we see that the process of data collection can be modeled as an M/D/1 queue with deadlines. We are not aware of a general expression characterizing the data loss fraction for this queue. However, such an expression can be found if the network is dense ($N \to \infty$). As a consequence of this queuing formulation, data collection can be analyzed independent of quantities that are hard to characterize, such as the shape of the network and the path of the observer. It is interesting to note that all the information that we need about these quantities is contained in the parameter $R$ defined in (4). Therefore, the queuing formulation reduces a complex network power optimization problem to a simpler, tractable, and more familiar one. We will also see in Section 5 that asymptotic analysis using the queuing formulation gives us analytical expressions for data loss in a dense WSN. We conclude our discussion of the queuing analogy here. We are now in a position to discuss power minimization using the queue.

### 4.2 Power Minimization

Our goal is to find the pair $(R_m^*, D^*)$ that achieves data loss below a specified fraction $F_{\max}$ with the minimum $P$.[9] In other words, we are trying to solve a constrained minimization problem with $R_m$ and $D$ as our variables, an upper bound on $F(R_m, D)$ as the constraint, and $P(R_m, D)$ as our objective function. The value of $F$ can be estimated from the average fraction of arrivals dropped by the queue. In the absence of an analytical expression for $F(R_m, D)$, the power optimization procedure will involve identifying pairs $(R_m, D)$ (through a queuing simulation) for which the data loss fraction is below $F_{\max}$, followed by choosing the pair among these that minimizes $P$. Uniqueness of the minimum is hard to prove due to the lack of an analytical characterization of $F$. However, the empirical behavior of $F$ with $R_m$ and $D$ together with the convexity of $P$ suggests uniqueness of the minimum. A procedure to find the minimum power solution is provided below, and it is used in an example in Section 4.3.

(1) $D \leftarrow (1 - F_{\max})D_{net}$.

(2) $R_m \leftarrow \sqrt{((1 - F_{\max})R)^2 + (vT_{cycle}D_{sens}/2D)^2}$.

(3) `until` $(F(R_m, D) < F_{\max})$[10]
`increment` $R_m$ `in small steps.`

(4) `calculate` $P_{net}(R_m, D)$.
`if (a power minimum has been reached)`
`then (output the current values of` $D$, $R_m$, and $P_{net}(R_m, D)$`)`
`else (increment` $D$ `by a small amount` $\delta$ `and go back to step 2).`

The initial values of $D$ and $R_m$ above correspond to the following lower bounds:

$$D \geq (1 - F_{\max})D_{net}, \tag{25}$$

$$R_m \geq \sqrt{((1 - F_{\max})R)^2 + (vT/2)^2} \geq (1 - F_{\max})R, \tag{26}$$

which are proved in Section 5.

[9] It can be seen from (6) that minimizing $P$ is the same as minimizing $P_{net}$ when $F$ is fixed.
[10] A queuing simulation is necessary to determine $F(R_m, D)$ for each value of $R_m$.

Table I. List of System Parameters

| Parameter | Value |
|---|---|
| $F_{\max}$ = maximum allowable data loss fraction | 0.2 |
| $\gamma$ = path attenuation of wireless channel | 2, 3, 4, 5, 6 |
| $R$ = network parameter related to range | 100 m |
| $A$ = area covered by the WSN | 4 sq km |
| $v$ = observer speed | 10 m/s |
| $N$ = number of nodes | 100 |
| $B$ = bandwidth of the system | 100 kHz |
| $N_0$ = noise power spectral density | $10^{-19}$ W/Hz |
| $D_{sens}$ = rate at which a sensor collects data | 1k/bs |
| $D_{net}$ = rate at which the network collects data | $ND_{sens}$ |
| $T_{cycle}$ = time needed to complete one cycle | $A/(2Rv)$ |
| $D_{cycle}$ = data collected by a node in one cycle | $D_{sens} * T_{cycle}$ |

### 4.3 Power Minimization—A Numeric Example

We now present an example of network power minimization. Table I lists the parameters used in this example.

The queue is simulated for the parameters in Table I. Figure 4 is a contour plot showing how data loss depends on $R_m$ and $T$ (equivalently $D$) in this network. The tradeoff between range $R_m$ and data rate $D$ is evident from this plot. A long range compensates for low data rate (large $T$), and similarly high data rate (small $T$) compensates for short range, to reach the same value of $F$ in each case. Also note that this tradeoff is effective only within the limits specified by (25) and (26). No range, however large, can attain a given value of $F_{\max}$, if $D$ does not satisfy (25). Similarly, to attain a given $F_{\max}$, $R_m$ must satisfy (26). These bounds corresponding to $F_{\max} = 0.2$ are marked in Figure 4.

Next, we calculate $P_{net}$ for pairs $(R_m, D)$ that achieve $F \leq 0.2$. This corresponds to points on or below the contour for $F = 0.2$ in Figure 4. Figure 5 shows $P_{net}$ as a function of $R_m$ for different values of the channel attenuation $\gamma$. It is implicit that, for each $R_m$, the smallest value of $D$ necessary to ensure $F \leq 0.2$ is chosen. It can be seen that the network power attains its minimum at a value of $R_m$ which is higher than the asymptotic lower bound of 80 m calculated from (26). Finally, note that $\gamma$ does not seem to affect the power minimum significantly.

We will prove in Section 5 that, in a dense network, the power minimum is attained when $D$ and $R_m$ are both equal to their respective lower bounds. But this is clearly not true in a sparse network, as seen from Figures 6 and 7. The plots show the values of $R_m$ and $D$ which correspond to minimum $P_{net}$ for each of four different values of $N$. The network parameters (except $N$) are same as in Table I. In a sparse network with 30 nodes in a 4 sq km area, the minimum power range is 150 m, which is nearly double the asymptotic lower bound of $(1 - F_{\max})R = 80$ m, and the corresponding data rate is nearly four times its asymptotic lower bound of $(1 - F_{\max})D_{net} = 80$ kb/s. We also see that the minimum power range tends to the asymptotic lower bound of 80 m, and the corresponding data rate becomes close to 80 kb/s as $N$ becomes large. As $N \to \infty$, the minimum power range and data rate would actually equal the respective lower bounds, but the fact that we are not far from the

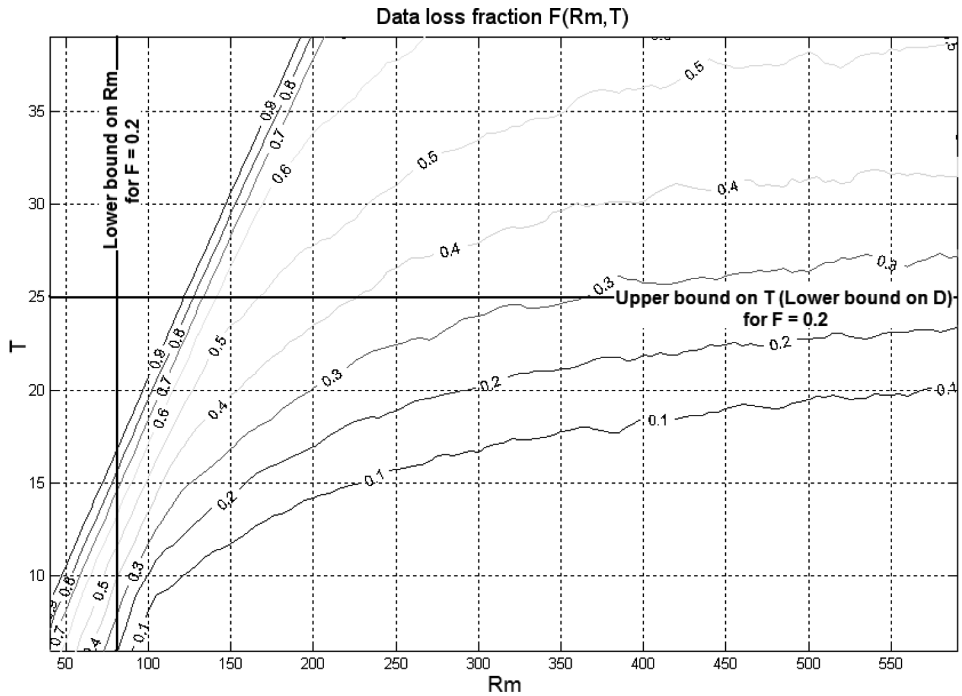


Fig. 4. A contour plot showing the data loss pattern for different choices of $R_m$ and $T$.

lower bound even with $N = 300$ is of empirical significance. It indicates that asymptotic results can give fair estimates of the optimum range and data rate at realistic node densities. Also note from Figure 6 that, for large communication range, the network power remains same irrespective of $N$. The fact that data is being received from nodes in discrete bursts is no longer significant with the smoothing effect of a large communication range.

## 4.4 Communication Without Data Loss

Until now, our discussion has concentrated on the tradeoff between data loss and power. But certain applications may not tolerate data loss at all. For such applications, we are interested in deriving conditions that can deterministically ensure that there will be no data loss even when the observer is mobile. It is possible to ensure zero data loss by increasing the communication range or the data rate to large values. However, such solutions have little or no advantage over stationary observer WSNs in terms of power.

As we have discussed, the independent distribution of sensors can lead to high local node density, and cause data loss. This suggests that if a minimum separation between sensors can be ensured (not an independent distribution any more), then zero data loss is a possibility. This is indeed true, and the result is quantified in the following theorem.

THEOREM 4.6. *No data is lost on account of observer mobility if the communication range $R_m$ satisfies* (5)*, and the minimum distance of separation $d$*

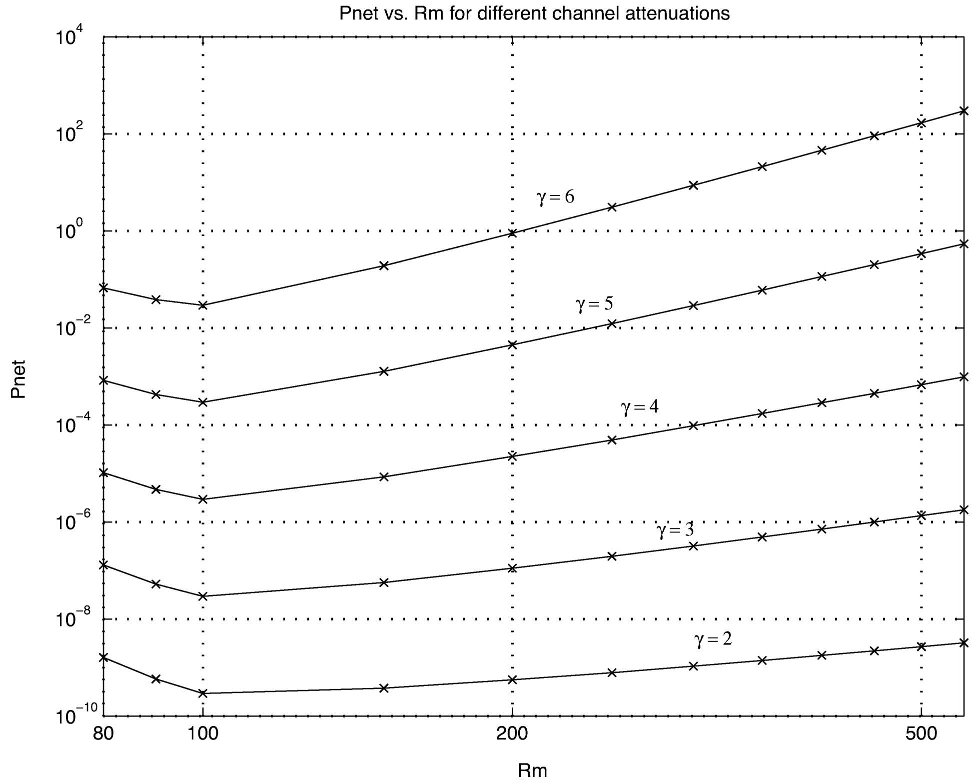


Fig. 5. Variation of network power with $R_m$ for different channel attenuations.

*between all pairs of nodes obeys the condition*

$$d \geq \sqrt{(2R)^2 + (vT)^2}. \tag{27}$$

PROOF. See Appendix A □

The above theorem applies to situations where some amount of control can be exercised on the positioning of sensors, or alternatively, if data from sensors that are too close to each other is redundant enough to be ignored.

Recall from Section 2.3 that (5) is a necessary but not sufficient condition for ensuring zero data loss. Condition (27) is sufficient for preventing data loss, but unlike (5), it is not necessary. It is possible that (27) is violated but the path of the observer is such that no data is lost. However, (27) is tight in the sense that if (5) is a strict equality, then, for every $d' < d$, it is possible to arrange sensors and to choose a path such that data loss is inevitable. An example demonstrating the above tightness property is provided in Appendix A.

## 5. ASYMPTOTIC ANALYSIS OF POWER-DATA LOSS TRADEOFF

In this section, we will derive expressions characterizing the tradeoff between power and data loss in the case of a dense network ($N \to \infty$). As $N \to \infty$,

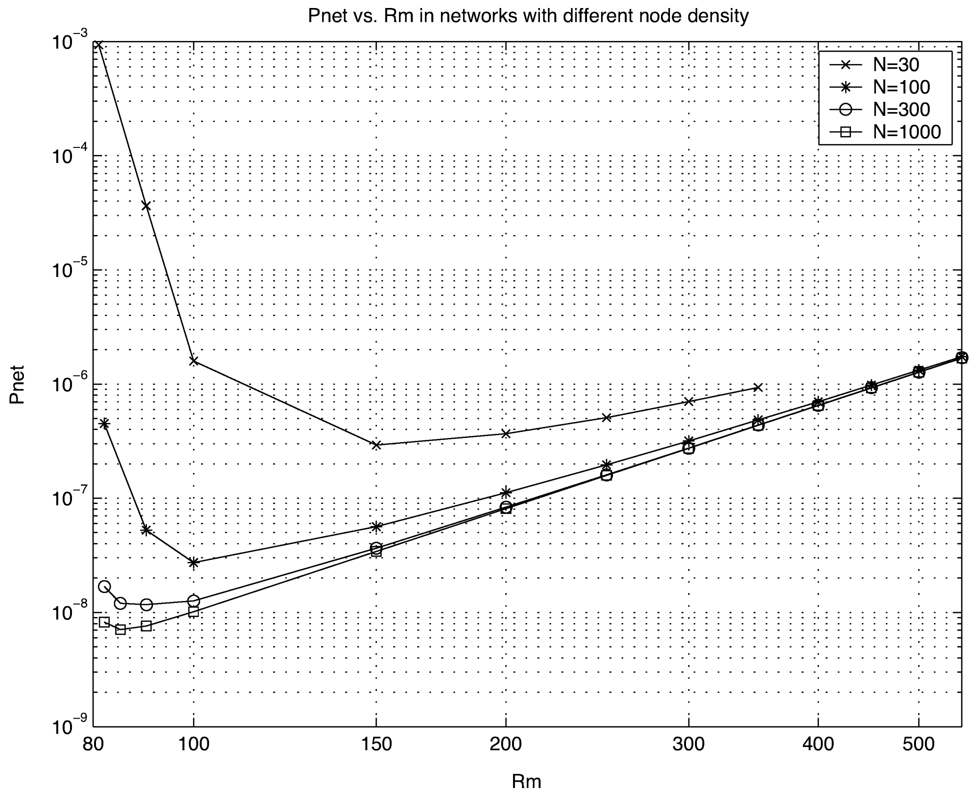


Fig. 6. Network power versus $R_m$ for different node densities.

individual sensors collect information at a rate $D_{sens} = D_{net}/N$, which goes to 0, as does the time $T$ taken by a sensor to transfer its information to the observer. Our goal, as before, is to find the pair $(R_m^*, D^*)$ that minimizes $P$ while limiting data loss below a specified fraction $F_{\max} > 0$. In this direction, we present the following theorem.

THEOREM 5.1. *As $N \to \infty$, for any given $F_{\max} > 0$, the pair $(R_m^*, D^*)$ that minimizes $P(R_m, D)$ is given by*

$$\lim_{N\to\infty} R_m^* = \sqrt{((1 - F_{\max})R)^2 + (vT/2)^2} \simeq (1 - F_{\max})R, \tag{28}$$

$$\lim_{N\to\infty} D^* = (1 - F_{\max})D_{net}. \tag{29}$$

PROOF. See Appendix B. □

COROLLARY 5.2.

$$\lim_{N\to\infty} (1 - F_{\max})NT = T_{cycle}. \tag{30}$$

PROOF. Using (11), and Lemma B.1. □

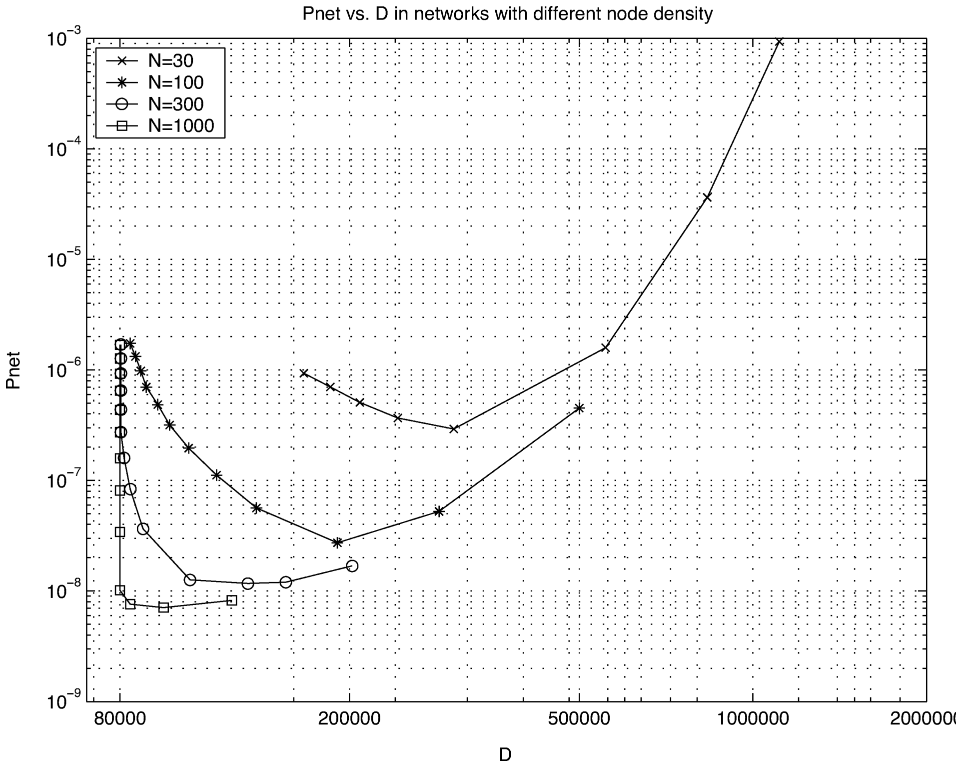


Fig. 7. Network power versus $D$ for different node densities.

Corollary 5.3. *The average network power in a dense network is*

$$P_{net} = ((1 - F_{\max})R)^{\gamma}\left(2^{(1-F_{\max})D_{net}} - 1\right). \tag{31}$$

Proof. Using (10), (6), and Theorem 5.1. □

Thus we see that, in a dense network, the network power can be expressed analytically in terms of the system parameters $R$ and $D_{net}$ and the specified data loss fraction $F_{\max}$.

Theorem 5.1 is a consequence of the law of large numbers. In a dense network, even minor variations in local node density are extremely improbable. As a result, sensor arrivals at the observer are almost uniform, and the observer can serve all sensors in range with high probability if the sensors have nonzero laxity. From the proof, it can be observed that the exact nature of the distribution of deadlines is irrelevant in a dense network, so long as nodes have nonzero laxity with high probability.

## 6. POWER COMPARISION WITH STATIC WSNS

The power consumption in a mobile observer WSN depends on the area and the shape of the WSN, as well as the path of the observer. The influence of these quantities manifests through the parameters $A$ and $R$. Power savings due to mobility depend on these parameters. In this section, we illustrate the impact of mobility in two representative scenarios. Analyzing the first case gives us an appreciation of the amount of savings in communication power that one might expect in a typical neighborhood WSN. The second case demonstrates the role played by the parameter $R$ in power savings, and also throws light on issues of practical importance such as the influence of network edges, and factors that govern the choice of path, if there is a choice.

The networks are assumed to be dense in each case. We consider networks shaped like planar discs. In the case of a static observer, we assume that the observer is positioned at the center of the disc. For fair comparison, we require that in both the static and mobile observer networks, only a fraction $(1 - F)$ of the sensors send data to the observer, and all sensors use the same range and data rate.

### 6.1 Case 1: A Neighborhood WSN

The expression for network power in a dense mobile observer network is given by (31), which becomes

$$P_{mobile} = BN_0((1 - F_{\max})R)^{\gamma} \left(2^{(1-F_{\max})ND_{sens}/B} - 1\right) \tag{32}$$

on taking into account the bandwidth $B$ and the noise power spectral density $N_0$.

Now suppose that the same WSN has a centrally placed static observer. In order for a fraction $(1-F_{\max})$ of nodes to be within range, the range must equal $\sqrt{1 - F_{\max}}$ times the radius of the network. The rate of communication equals the rate of data collection by $(1 - F_{\max})N$ sensors, which is $(1 - F_{\max})ND_{sens}$. From these observations, the network power, which is the sum of individual node powers, is found to be

$$P_{static} = BN_0 \left(\sqrt{\frac{(1 - F_{\max})A}{\pi}}\right)^{\gamma} \left(2^{(1-F_{\max})ND_{sens}/B} - 1\right) \tag{33}$$

for the static observer scenario.

From (32) and (33), we see that

$$\frac{P_{mobile}}{P_{static}} = \left(\frac{\sqrt{\pi(1 - F_{\max})}\, R}{\sqrt{A}}\right)^{\gamma}. \tag{34}$$

Thus, for a fixed network area $A$, and a specified data loss fraction $F_{\max}$, the ratio of power consumptions depends only on $R$. The distance $R$ is a measure of the extent to which the path reaches different parts of the network. A dense path (such as a space filling curve) will lead to a small value of $R$, and therefore significant power savings.

If a typical neighborhood WSN has an area $A = \pi(1)^2$ sq km, and the observer makes use of a local vehicle that carries it to within $R = 0.1$ km of each sensor, then from (34) we can see that the network communication power is reduced by at least two orders of magnitude (since $\gamma \geq 2$). This comes as no surprise since the required communication range is reduced by a factor of 10. Even from this rough estimate, it is clear that in wide area WSNs, where the communication

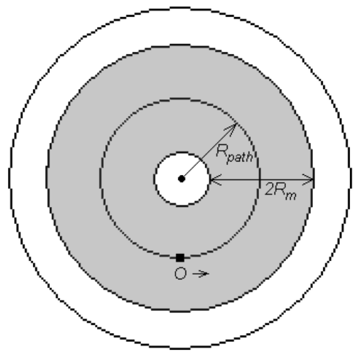


Fig. 8. A disk network with a circular path of radius $R_{path}$. Only sensors in the shaded region are able to communicate with the observer $O$.

power constitutes a large percentage of the total power, deterministic mobility can aid in reducing communication power considerably.

### 6.2 Case 2: Disk Network with Circular Closed Path

This example evaluates power savings in a disk-shaped network with a circular path that is concentric with the network. We will study how power savings change with the radius of the path. Apart from that, this example will also throw light on edge effects and the consequences of a mobility model mismatch.

We begin by examining the path itself. A circular path in a circular network may not fit our mobility model in several situations. First, if the radius is small, it will be inaccurate to treat the path as a straight line over distances of the order of the communication range. Second, Equation (1) will be valid only if the path is of exactly half the radius as the network. Third, a path with large radius (nearly equal to the network radius) will require a large range to collect data from nodes near the center of the network. This range will be large enough to collect data from points outside the network, or, in other words, the network edge will come into picture. But, as we will see, even though the path may not fit our mobility model, it can still save power irrespective of its radius. First, note that the arrival rate will be uniform due to the symmetry of the path, and because the network is dense. Second, since the network is dense, $T \to 0$, and therefore almost all nodes in range will remain in range long enough to receive service. In a dense network, the data rate is given by (29). The choice of communication range is dictated by the requirement that a fraction $(1 - F_{\max})$ of sensors remain within range. This boils down to ensuring that the set of points that are within range $R_m$ from the path of radius $R_{path}$ constitute an area $(1 - F_{\max})A$ (see Figure 8). From the above criterion, the minimum $R_m$ necessary for $(1 - F_{\max})N$ nodes to be in range is found to be

$$R_m = \begin{cases} \sqrt{1 - F_{\max}}\, R_{net} - R_{path}, & 0 \le R_{path} \le \frac{\sqrt{1-F_{\max}}\, R_{net}}{2}, \\ \frac{(1-F_{\max})R_{net}^2}{4R_{path}}, & \frac{\sqrt{1-F_{\max}}\, R_{net}}{2} < R_{path} < \frac{(1+\sqrt{F_{\max}})\, R_{net}}{2}, \\ R_{path} - \sqrt{F_{\max}}\, R_{net}, & \frac{(1+\sqrt{F_{\max}})\, R_{net}}{2} \le R_{path} \le R_{net}. \end{cases}$$

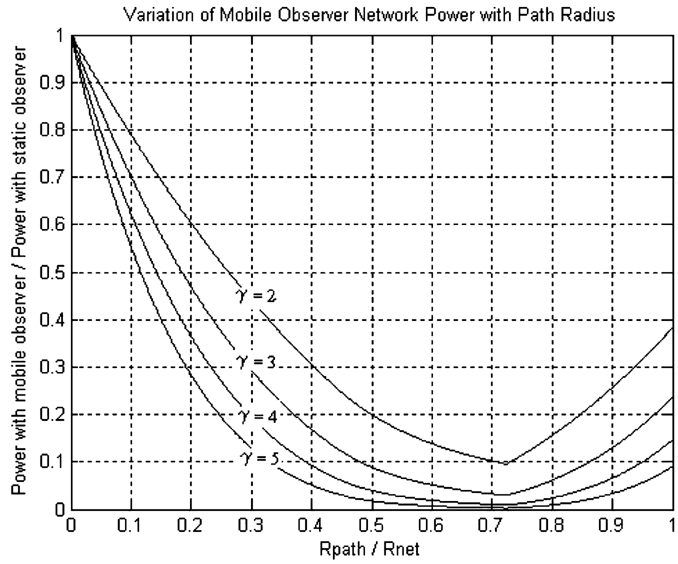


Fig. 9. Role of $R$ in determining power savings.

Using the above, we plot the mobile observer network power (normalized with the static observer network power) corresponding to $F_{\max} = 0.2$ for different $R_{path}$ and $\gamma$ in Figure 9. The plot reiterates that power savings depend on $R_{path}$. As $R_{path}$ increases from zero, the range $R_m$ necessary to cover the same area decreases, thereby saving power. Power savings are maximized when $R_{path} = \frac{(1+\sqrt{F_{\max}})\ R_{net}}{2}$. For larger values of $R_{path}$, edge effects degrade performance. By edge effect, it is implied that the edge of the network is inside communication range of the observer. When this happens, the observer stops receiving data from some areas that are within range (but outside the network). In other words, a longer range is necessary to receive data from the same number of sensors, leading to increased power consumption.

## 7. CONCLUSIONS AND FUTURE WORK

We have demonstrated the feasibility of using a mobile observer to save communication power in a WSN. Not only does the use of a mobile observer save overall network power, it also equates the power consumption at different nodes, thereby ensuring that the network will not collapse due to localized node failures.

There are challenges in using a mobile observer that need to be studied further. We have explored the case of a single mobile observer that moves on a predictable route with at a constant speed. Extensions to other mobility patterns need to be explored. Research must also look into mobile observer protocols; nodes which collect information at different rates; multiple mobile observers; heterogeneous networks where not all nodes are power-limited; and a combination of mobility and multihop communication.

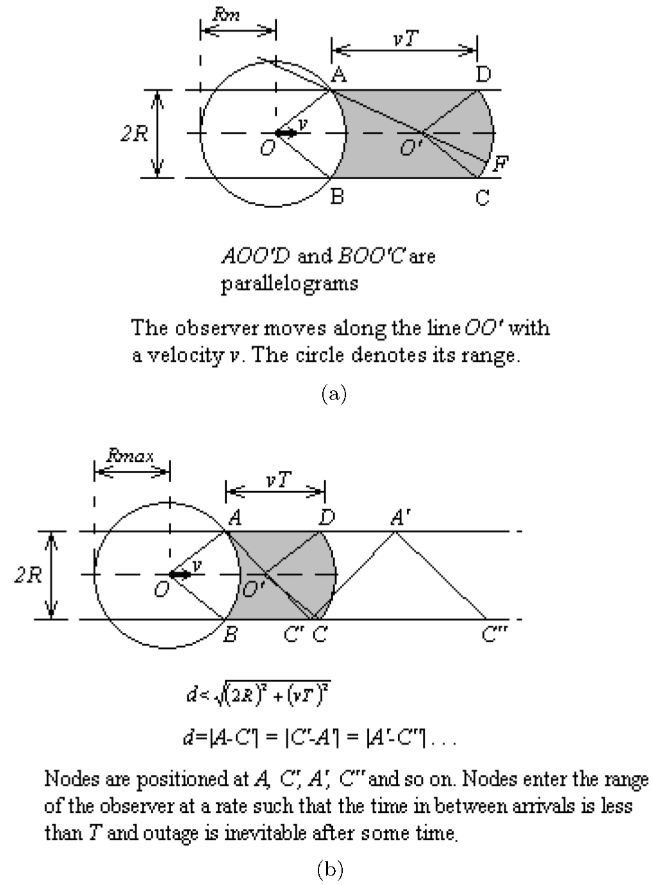


Fig. A. 1.

## APPENDIX

### A. CONDITION FOR GUARANTEEING ZERO DATA LOSS

The sufficient condition for ensuring zero data loss with minimum separation of nodes $d$ is derived as follows (see Figure A.1(a)). In this proof, we will assume that (5) is satisfied with equality. It is obvious that the proof will also be valid for larger communication ranges (if (5) is a strict inequality). Data transfer from a node to the observer takes time $T$. If we can ensure that the entry of two nodes into the range of the observer is spaced in time by $T$, then data loss will not occur. Thus, if a sensor node is placed anywhere on the arc $\widehat{AB}$ (see Figure A.1(a)), then no other node can lie within the shaded region $ABCD$. This implies that the minimum separation $d$ must be greater than or equal to the distance from any point on arc $\widehat{AB}$ to any other point on the boundary of $ABCD$, that is,

$$d \geq \max(|x - y|), \quad x \in \widehat{AB}, \; y \in ABCD. \tag{35}$$

Let $x = U$ and $y = V$ be the pair for which $|x - y|$ is maximized.

THEOREM A.1. *$U = A$ and $V = C$ (or $U = B$ and $V = D$) are the pair that maximizes $|x - y|$.*

PROOF. The result follows from a series of claims. The claims themselves are almost trivial to prove; therefore for brevity we will omit their proofs here. □

CLAIM A.2. *Choose and fix an arbitrary point $x = P \in \widehat{AB}$. Then, the point $y = Q \in ABCD$, for which $|P - y|$ is maximized does not lie on the line segments $BC$ or $AD$ except possibly that $Q$ is one of the end points $A, B, C$ or $D$.*

COROLLARY A.3. *Since this claim holds for arbitrary $x$, it must hold for $x = U$, the point which achieves the overall maximum. Therefore, $V \in \{\widehat{AB}, \widehat{CD}\}$.*

CLAIM A.4. *Choose and fix an arbitrary point $x = P \in AB$. Suppose that the point $y = Q \in ABCD$ is the point for which $|P - y|$ is maximized. Then, unless $P$ is one of the end points of $\widehat{AB}$, $|P - Q| < |U - V|$. In other words, $U \in \{A, B\}$.*

COROLLARY A.5. *$V$ lies on $\widehat{CD}$, for if $V$ were to lie on $\widehat{AB}$, by moving horizontally to the corresponding point on $\widehat{CD}$, one could show that this point is farther from $U$ than $V$ is.*

The problem is therefore reduced to that of finding $V$ from the set of points on $\widehat{CD}$. $U$ has been ascertained to be either $A$ or $B$ (which one we choose makes no difference). Consider the line passing through $A$ and $O'$. This line cuts the arc $\widehat{CD}$ if and only if

$$(2R)^2 + (vT)^2 > (2R_m)^2, \tag{36}$$

which contradicts (5). The case of interest is when the line passing through $A$ and $O'$ does not cut $\widehat{CD}$. This happens when $A$ lies within the circle centered at $O'$ and having radius $R_{\max}$. We prove the following claim for this situation:

CLAIM A.6. *When the line passing through $A$ and $O'$ does not cut $\widehat{CD}$, then*

$$|A - C| = \max(|x - y|), \quad x \in AB, y \in ABCD. \tag{37}$$

This also proves our original claim that $U = A$ and $V = C$ (or $U = B$ and $V = D$) are the pair that maximizes $|x - y|$. □

This is the result using which we obtain a meaningful relationship between the minimum separation $d$ and our system parameters. Since

$$|A - C| = |B - D| = \sqrt{(2R)^2 + (vT)^2}\ , \tag{38}$$

we conclude that

$$d \geq \sqrt{(2R)^2 + (vT)^2} \tag{39}$$

is a sufficient condition to guarantee zero data loss.

The above condition is not necessary to avoid data loss, meaning that it may be violated but data may not be lost. However, an interesting point to note is that, if this condition is not satisfied, then there exist bad arrangements of sensors that lead to data loss. Figure A.1(b) shows one such arrangement of sensors where data loss is unavoidable with

$$d < \sqrt{(2R)^2 + (vT)^2}\,. \tag{40}$$

## B. PROOF OF THEOREM 5.1

The following is a sketch of the proof. We know that power is monotonically increasing with $R_m$ as well as $D$. We will prove this theorem by showing the following:

(1) Data loss exceeds $F_{\max}$ if either $R_m < R_m^*$ or $D < D^*$.

(2) Data loss does not exceed $F_{\max}$ if $(R_m, D) = (R_{m\epsilon}^*, D^*)$, where[11]

$$R_{m\epsilon}^* = \sqrt{((1 - F_{\max})R(1+\epsilon))^2 + (vT/2)^2} \tag{41}$$

for any given $\epsilon > 0$ and for sufficiently large $N$.

It is easy to see that the data rate $D$ must equal or exceed $D^*$ for data loss to remain below $F_{\max}$. From (2) and (11), we have

$$D = \frac{(1-F)D_{net}}{f_b} \geq (1-F)D_{net} \geq (1-F_{\max})D_{net} = D^*. \tag{42}$$

Intuitively, (42) means that the rate $D$ at which data is transferred must equal or exceed the rate at which it is collected by the requisite fraction $1 - F_{\max}$ of sensors.

It is also easy to see that $R_m$ cannot be less than $R_m^*$ for the data loss fraction to be less than $F_{\max}$. Assume for a contradiction that $R_m < R_m^*$. Then, since $R_m$ does not satisfy (5), from (19) and (1) the arrivals at the observer are Poisson-distributed with a mean arrival rate of $2vN\sqrt{R_m^2 - (vT/2)^2}/A = N\sqrt{R_m^2 - (vT/2)^2}/RT_{cycle}$. Therefore:

$$\begin{aligned}\text{Average number of arrivals per cycle} &= \frac{N\sqrt{R_m^2 - (vT/2)^2}}{RT_{cycle}}T_{cycle} &(43)\\ &< \frac{N\sqrt{{R_m^*}^2 - (vT/2)^2}}{R} &(44)\\ &= (1-F_{\max})N, &(45)\end{aligned}$$

where the inequality follows from our assumption that $R_m < R_m^*$, and the last step follows from the definition of $R_m^*$.

Hence, the average number of arrivals falls short of $(1 - F_{\max})N$. Even if all arrivals are successfully serviced, the data loss fraction will still be in excess of $F_{\max}$, contradicting our assumption that $R_m < R_m^*$ suffices to keep the data loss fraction below $F_{\max}$.

It remains to be shown that for any $\epsilon > 0$, $(R_m, D) = (R_{m\epsilon}^*, D^*)$ limits data loss to $F_{\max}$. The following result will be necessary for this:

[11]We wish to define a quantity that is infinitesimally larger than $R_m^*$. Using $R_m^* + \epsilon$ makes subsequent steps cumbersome; therefore we define $R_{m\epsilon}^*$.

For any $\epsilon > 0$, if $(R_m, D) = (R^*_{m\epsilon}, D^*)$, then

$$\lim_{N\to\infty} f_b = 1. \tag{46}$$

The above result will be the most crucial link in substantiating our claim in (28). For the sake of continuity, let us assume that (46) holds. Then, observe that

$$D = \frac{(1 - F(R_m, D))D_{net}}{f_b} \Rightarrow F(R_m, D) = 1 - \frac{Df_b}{D_{net}} \tag{47}$$

$$\Rightarrow F(R^*_m, D^*) = 1 - (1 - F_{\max})f_b \tag{48}$$

$$\Rightarrow \lim_{N\to\infty} F(R^*_{m\epsilon}, D^*) = F_{\max}, \tag{49}$$

which completes our proof. Here, (47) follows from (42), (48) from (29), and (49) from (46).

It is now time to prove claim (46).

LEMMA B.1. $\lim_{N\to\infty} f_b = 1$.

PROOF. We define the following events: □

—$E_1$. The observer is idle at a randomly chosen instant $t$.

—$E_2$. All sensor arrivals in $(-\infty, t]$ have either been serviced completely before $t$, or they are past their laxity.

—$E_3$. All sensor arrivals in $(t - c/\sqrt{N}, t]$ have either been serviced completely before $t$, or they are past their laxity. Here, $c$ is a constant.

It should be clear from the definitions, and from the fact that the scheduling policy is nonidling, that $E_1 = E_2$, and $E_2 \subseteq E_3$. Thus,

$$P(E_3) \geq P(E_2) = P(E_1) = 1 - f_b. \tag{50}$$

Now consider the event $E_3$. It is easy to see from (22) that the probability that an arrival in $(t - c/\sqrt{N}, t]$ will be past its laxity at $t$ is negligibly small, being at most $O(1/\sqrt{N})$. Therefore, for $E_3$ to be true, (nearly) all arrivals in $(t - c/\sqrt{N}, t]$ must be serviced completely by $t$. From (2) and (42), the service time per arrival is

$$T = \frac{T_{cycle}D_{net}}{ND} = \frac{f_b T_{cycle}}{(1 - F)N}. \tag{51}$$

Therefore, the maximum number of arrivals that can be serviced in a duration $\frac{c}{\sqrt{N}}$ is

$$\frac{c}{\sqrt{N}T} = \frac{(1 - F)c\sqrt{N}}{f_b T_{cycle}}. \tag{52}$$

For $E_3$ to be true, the number of arrivals in $(t - c/\sqrt{N}, t]$ must not exceed the number that can be serviced in that duration. Hence,

$$P(E_3) \leq P\left(\text{number of arrivals in} \frac{c}{\sqrt{N}} \leq \frac{c}{\sqrt{N}T}\right). \tag{53}$$

Since the value of $R_m$ we are considering does not satisfy (5), the arrivals will be Poisson with a mean arrival rate of $\frac{2vN\sqrt{R_m^2-(vT/2)^2}}{A} = \frac{N\sqrt{R_m^2-(vT/2)^2}}{RT_{cycle}} \to \infty$ as $N \to \infty$. In the limit, as the arrival rate becomes large, the Poisson distribution becomes a normal distribution with the same mean and variance as the Poisson. Thus,

$$\frac{e^{-\mu}\mu^x}{x!} \approx \frac{1}{\sqrt{2\pi\mu}} \exp\left(-\frac{(x-\mu)^2}{2\mu}\right), \tag{54}$$

where

$$\mu = \left(\frac{\sqrt{R_m^2-(vT/2)^2}N}{RT_{cycle}}\right)\left(\frac{c}{\sqrt{N}}\right) = \frac{c\sqrt{N(R_m^2-(vT/2)^2)}}{RT_{cycle}}.$$

As a consequence of this,

$$\begin{aligned} P(E_3) &\le P\left(\text{number of arrivals in} \frac{c}{\sqrt{N}} \le \frac{c}{\sqrt{N}T}\right) \\ &= \sum_{x=0}^{\lfloor \frac{c}{\sqrt{N}T} \rfloor} \frac{e^{-\mu}\mu^x}{x!} \\ &\approx \int_{-\infty}^{\frac{c}{\sqrt{N}T}} \frac{1}{\sqrt{2\pi\mu}} \exp\left(-\frac{(x-\mu)^2}{2\mu}\right) dx \\ &= \frac{1}{2} + erf\left(\frac{\frac{c}{\sqrt{N}T} - \mu}{\sqrt{\mu}}\right) \\ &= \frac{1}{2} + erf\left(\frac{\frac{(1-F)c\sqrt{N}}{f_b T_{cycle}} - \frac{c\sqrt{N(R_m^2-(vT/2)^2)}}{RT_{cycle}}}{\sqrt{\frac{c\sqrt{N(R_m^2-(vT/2)^2)}}{RT_{cycle}}}}\right), \end{aligned} \tag{55}$$

where $erf(x)$ is the function

$$erf(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-(\frac{t^2}{2})} dt, \tag{56}$$

so that $erf(\infty) = 1/2$ and $erf(-\infty) = -1/2$. It is easy to see that, for $R_m = R_{m\epsilon}$,

$$\begin{aligned} \lim_{N\to\infty} \frac{\frac{(1-F)c\sqrt{N}}{f_b T_{cycle}} - \frac{c\sqrt{N(R_m^2-(vT/2)^2)}}{RT_{cycle}}}{\sqrt{\frac{c\sqrt{N(R_m^2-(vT/2)^2)}}{RT_{cycle}}}} &= \infty \quad \frac{1}{f_b} > 1+\epsilon \\ &= -\infty \quad \frac{1}{f_b} < 1+\epsilon. \end{aligned} \tag{57}$$

Now, if $f_b > 1/(1+\epsilon)$, then all is well and we find that (55) equals zero so that

$$\begin{aligned} 0 &\ge P(E_3) \ge P(E_2) = P(E_1) = 1 - f_b \\ &\Rightarrow f_b = 1. \end{aligned} \tag{58}$$

If, on the other hand, we assume that $f_b < 1/(1+\epsilon)$, then we can arrive at a contradiction by showing that $F = 0$ contrary to our assumption that $F_{\max} > 0$. Physically, this would mean that the system is working at higher power than necessary to ensure the specified $F_{\max}$; hence this is not the minimum.

For obtaining such a contradiction, consider an arrival at some arbitrary time $t - c/\sqrt{N}$. As argued before, the probability that it will be past its laxity at $t$ is negligibly small. We will show that this arrival will have received service with probability 1 if $f_b < (1-F)$, showing that there is no data loss ($F = 0$). Since the observer acts as a clairvoyant scheduler with perfect knowledge of node arrivals and deadlines, it will outperform any schedule in terms of data loss $F$. We will show that even with a nonpreemptive, nonidling LCFS (last-come first-served) schedule, $F = 0$ can be achieved. For convenience, we define the following event:

—[$E_4$] The arrival at $t - c/\sqrt{N}$ does not begin to receive service by $t$.

Since the schedule is nonidling, the number of arrivals in $(t - c/\sqrt{N}, t]$ must exceed the number of sensors that can be serviced in that duration for $E_4$ to be true. Hence,

$$\begin{aligned} P(E_4) &\leq P\left(\text{number of arrivals in} \frac{c}{\sqrt{N}} > \frac{c}{\sqrt{N}T}\right) \\ &= 1 - P\left(\text{number of arrivals in} \frac{c}{\sqrt{N}} \leq \frac{c}{\sqrt{N}T}\right) \\ &\approx \frac{1}{2} - erf\left(\frac{\frac{(1-F)c\sqrt{N}}{f_b T_{cycle}} - \frac{c\sqrt{N}}{T_{cycle}}}{\sqrt{\frac{c\sqrt{N}}{T_{cycle}}}}\right). \end{aligned} \tag{59}$$

If $f_b < (1-F)$, then (59) evaluates to 0 as $N \to \infty$, so that we have $P(E_4) = 0$. Thus an arbitrary arrival will receive service with probability 1, implying that the data loss fraction $F = 0$, which disagrees with our assumption that $F_{\max} > 0$.

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