15. Polarization

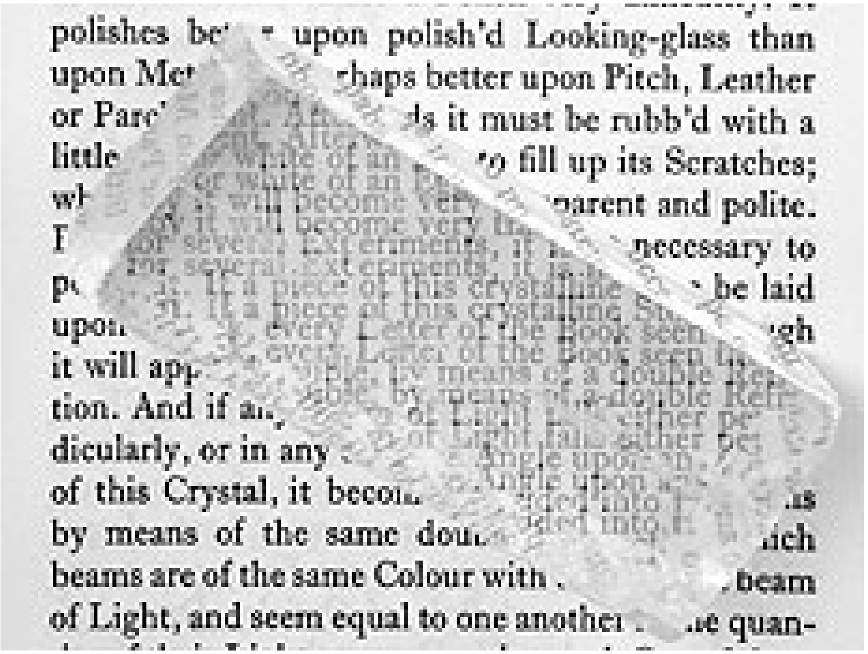
Linear, circular, and elliptical polarization

Mathematics of polarization

Uniaxial crystals

Birefringence

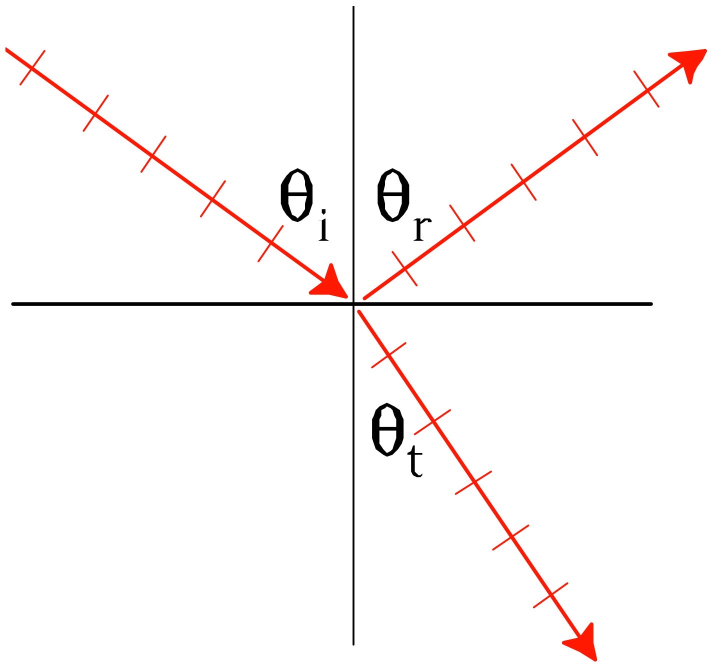
Polarizers

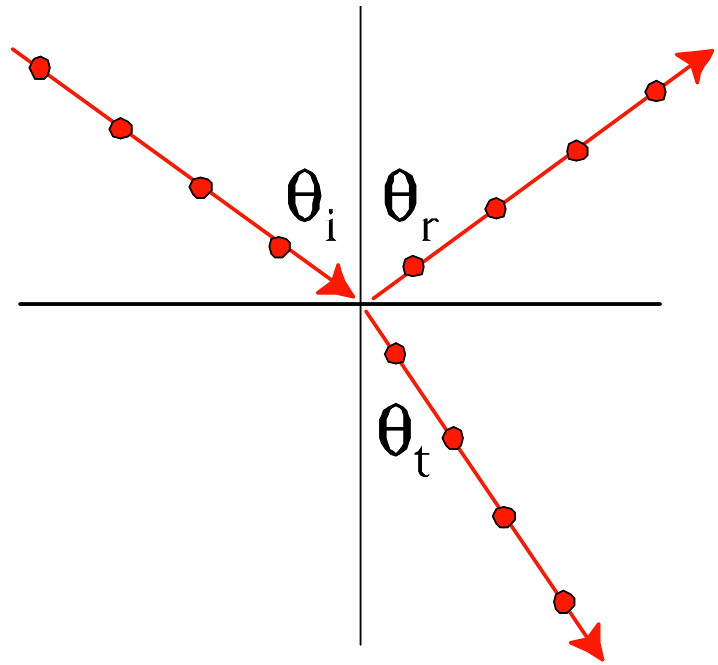


Polarization Notation near an interface

Parallel ("p") polarization

Perpendicular ("s") polarization





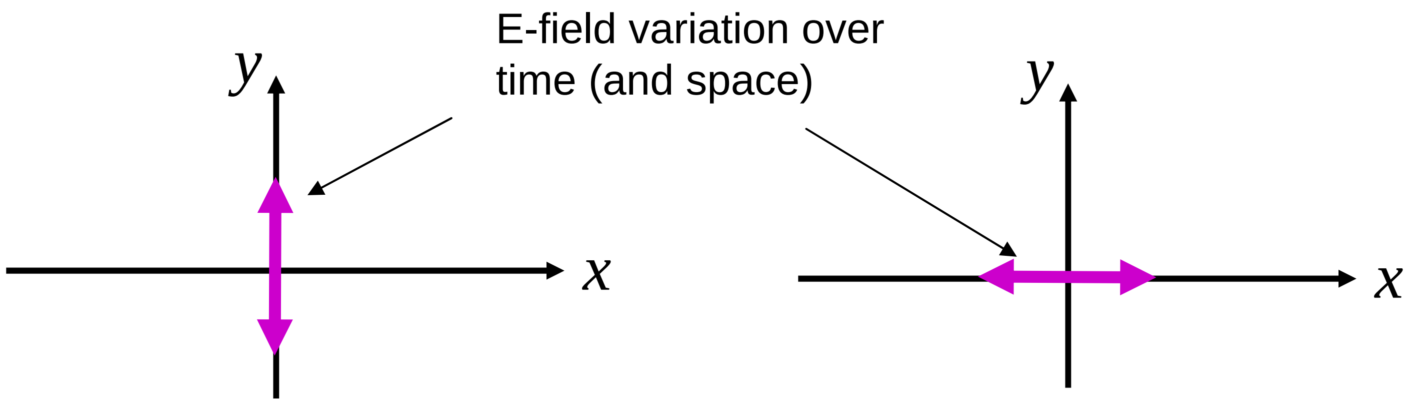
These are only defined relative to an interface between two media. But even when there is no interface around, we still need to consider the polarization of light waves.

Polarization of a light wave

We describe the polarization of a light wave (without any interface nearby) according to how the E-field vector varies in a projection onto a plane perpendicular to the propagation direction.

For convenience, the propagation direction is generally assumed to be along the positive z axis.

Here are two possibilities:



$$\vec{E}(\vec{r},t) = \underset{\sim}{E}_0 \hat{y} \exp\left[j(kz - \omega t) \right]$$

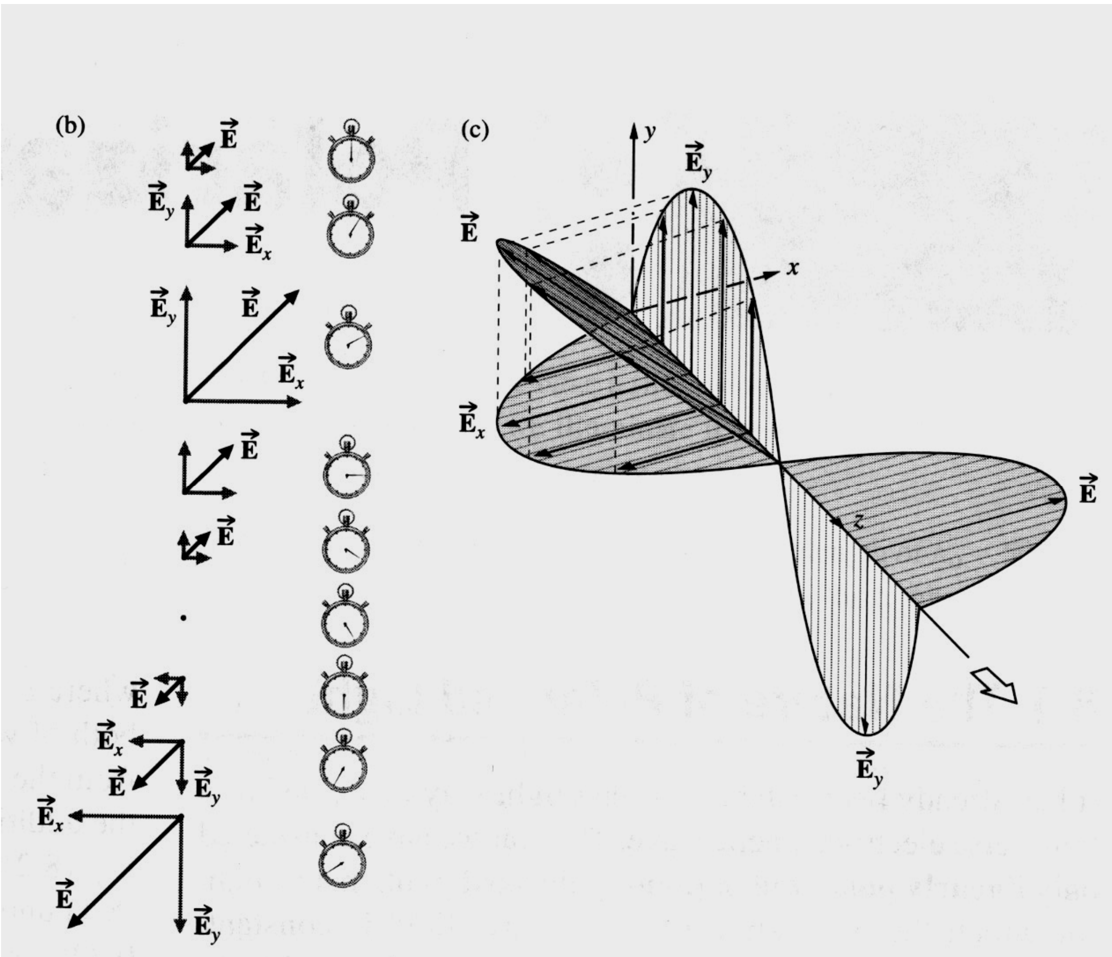
$$\vec{E}(\vec{r},t) = \underset{\sim}{E}_0 \hat{x} \exp\left[j(kz - \omega t) \right]$$

In these diagrams, the propagation direction is out of the page at you.

45° Polarization

$$E_x(z,t) = \text{Re}\left\{ \underset{\sim}{E}_0 \exp\left[j(kz - \omega t) \right] \right\}$$

$$E_y(z,t) = \text{Re}\left\{ \underset{\sim}{E}_0 \exp\left[j(kz - \omega t) \right] \right\}$$



Here, the complex amplitude, E_0 is the same for each component.

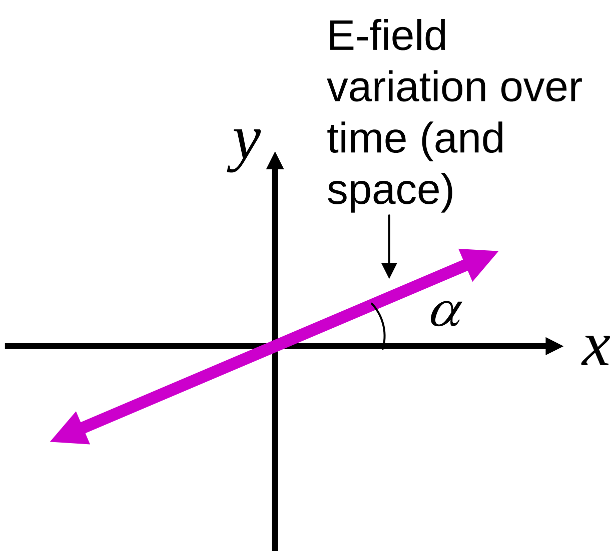
So the components are always in phase.

Arbitrary-Angle Linear Polarization

$$E_x(z,t) = \mathrm{Re}\left\{ \underset{\sim}{E}_0 \cos\left(\alpha\right) \exp\left[j(kz - \omega t) \right] \right\}$$

$$E_y(z,t) = \mathrm{Re}\left\{ \underset{\sim}{E}_0 \sin\left(\alpha\right) \exp\left[j(kz - \omega t) \right] \right\}$$

Here, the y-component is in phase with the x-component, but has different magnitude.



The Mathematics of Polarization

Define the polarization state of a field as a 2D vector—
"Jones vector" —containing the two complex amplitudes: $E = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$

For many purposes, we only care about the relative values:

$$\frac{E}{E_x} = \begin{bmatrix} 1 \\ E_y / E_x \end{bmatrix}$$

Specifically:

0° linear (x) polarization: $E_y/E_x = 0$ → $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

90° linear (y) polarization: $E_y/E_x = \infty$ → $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

45° linear polarization: $E_y/E_x = 1$

Arbitrary linear polarization: $\dfrac{E_y}{E_x} = \dfrac{\sin\alpha}{\cos\alpha} = \tan\alpha$

Jones vectors - a common mistake

NOTE: the Jones vector contains the complex amplitudes *only*. Its components *do not* depend on x,y,z, or t.

$$E = \begin{bmatrix} E_x e^{j(kz-\omega t)} \\ E_y e^{j(kz-\omega t)} \end{bmatrix}$$

← This is wrong!

Circular (or Helical) Polarization

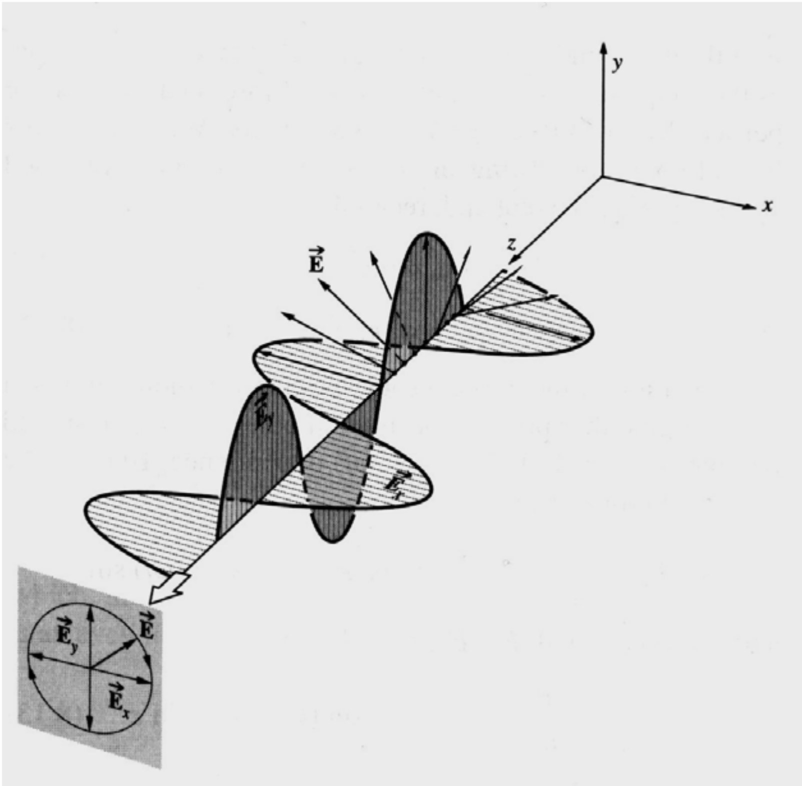
$$E_x(z,t) = E_0 \cos(kz - \omega t)$$
$$E_y(z,t) = E_0 \sin(kz - \omega t)$$

or, in complex notation:

$$E_x(z,t) = \mathrm{Re}\left\{ \underset{\sim}{E}_0 \exp\left[j(kz - \omega t) \right] \right\}$$
$$E_y(z,t) = \mathrm{Re}\left\{ -j\underset{\sim}{E}_0 \exp\left[j(kz - \omega t) \right] \right\}$$

Here, the complex amplitude of the y-component is $-j$ times the complex amplitude of the x-component.

So the components are always 90° out of phase.



The resulting E-field rotates clockwise around the k-vector (looking along k). This is called a right-handed rotation.

Right vs. Left Circular (or Helical) Polarization

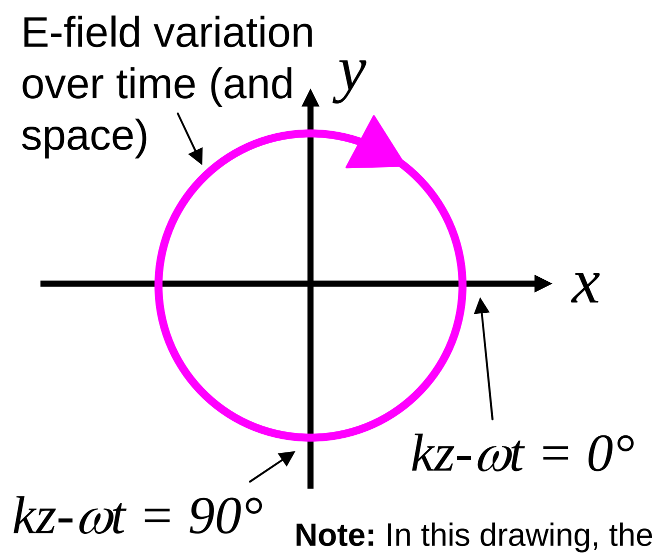
$$E_x(z,t) = E_0 \cos(kz - \omega t)$$
$$E_y(z,t) = -E_0 \sin(kz - \omega t)$$

or, more generally:

$$E_x(z,t) = \mathrm{Re}\left\{ \underset{\sim}{E_0} \exp\left[j(kz - \omega t) \right] \right\}$$
$$E_y(z,t) = \mathrm{Re}\left\{ +j\underset{\sim}{E_0} \exp\left[j(kz - \omega t) \right] \right\}$$

Here, the complex amplitude of the y-component is +j times the complex amplitude of the x-component.

So the components are always 90° out of phase, but in the other direction.

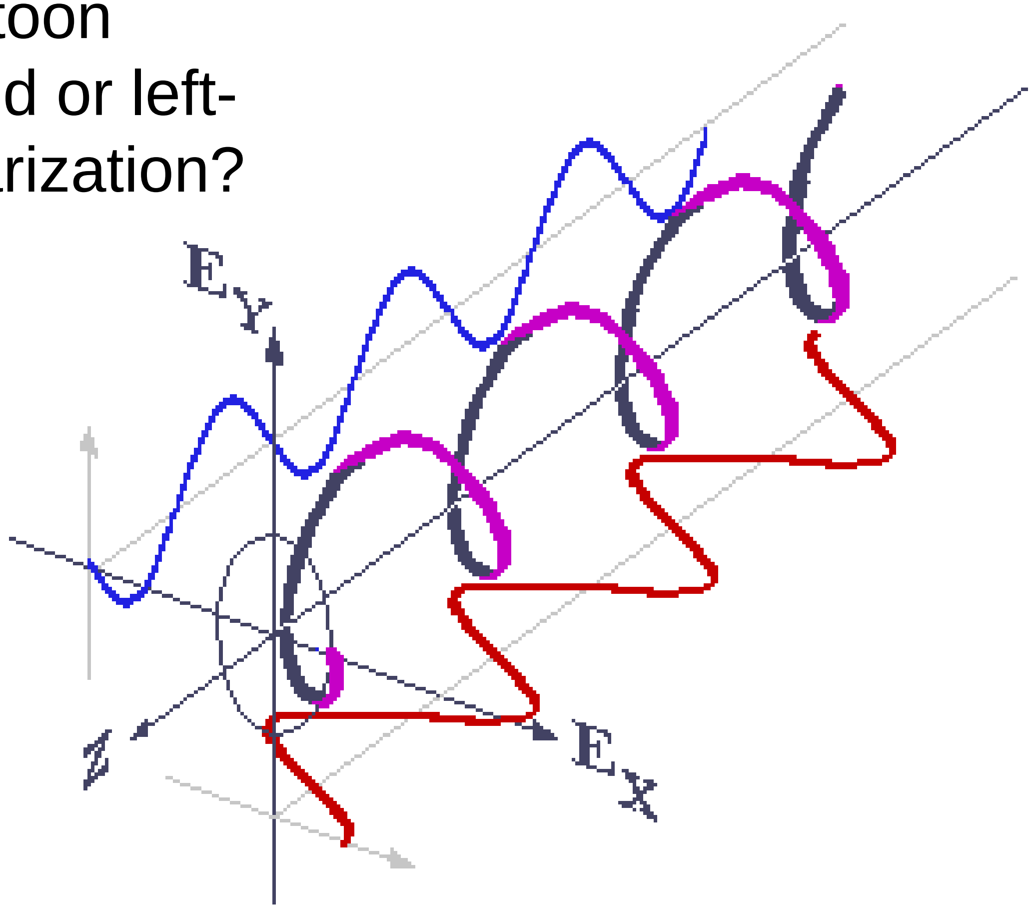


Note: In this drawing, the z axis is coming out of the screen at you. So you are looking in the *opposite* direction from the k-vector, which is why it rotates clockwise according to the arrow - but we refer to this as a counter-clockwise rotation.

The resulting E-field rotates counterclockwise around the k-vector (looking along k). This is a left-handed rotation.

Circular Polarization - the movie

Question: is this cartoon showing right-handed or left-handed circular polarization?

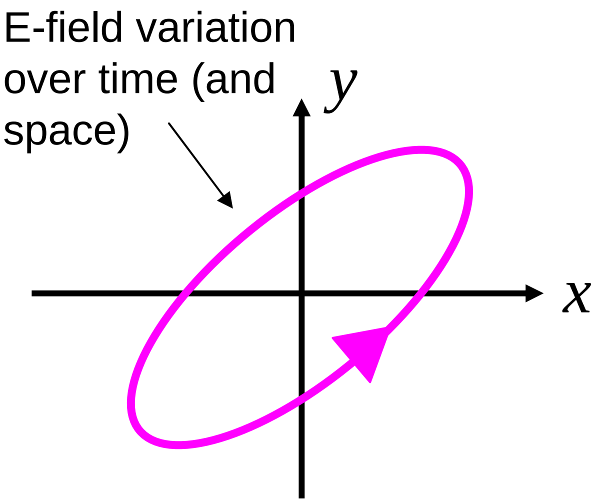


Unequal Arbitrary-Relative-Phase Components yield "Elliptical Polarization"

$$E_x(z,t) = E_{0x}\cos(kz - \omega t)$$

$$E_y(z,t) = E_{0y}\cos(kz - \omega t - \theta)$$

$$\text{where} \quad E_{0x} \neq E_{0y}$$



or, in complex notation:

$$E_x(z,t) = \mathrm{Re}\left\{ \underset{\sim}{E}_{0x} \exp\left[j(kz - \omega t) \right] \right\}$$

$$E_y(z,t) = \mathrm{Re}\left\{ \underset{\sim}{E}_{0y} \exp\left[j(kz - \omega t) \right] \right\}$$

where $\underset{\sim}{E}_{0x}$ and $\underset{\sim}{E}_{0y}$ are arbitrary complex amplitudes.

The resulting E-field can rotate clockwise or counter-clockwise around the k-vector.

The Mathematics of Circular and Elliptical Polarization

Circular polarization has an imaginary Jones vector y-component:

$$E = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 1 \\ \pm j \end{bmatrix}$$

Right circular polarization: $E_y / E_x = -j$ ← A clockwise rotation, when looking along the propagation direction.

Left circular polarization: $E_y / E_x = +j$ ← counterclockwise rotation.

For elliptical polarization, the two components have different amplitudes, and may even be complex:

$$E_y / E_x = a + jb$$

We can calculate the eccentricity and tilt of the ellipse if we feel like it.

An example

$$E = \begin{bmatrix} 1 \\ 1+j \end{bmatrix}$$

What is the polarization of this wave?

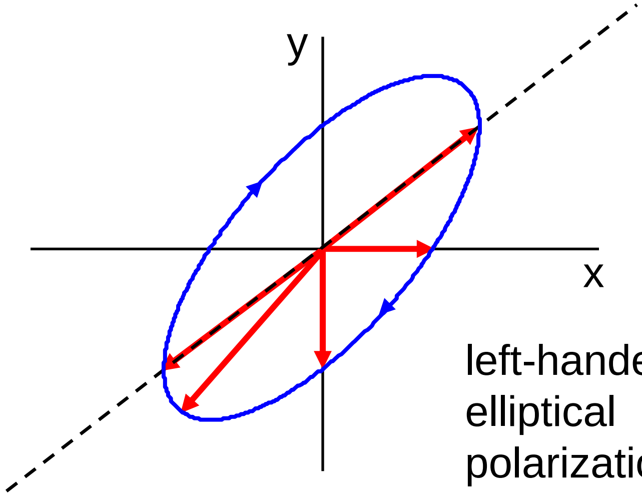
This Jones vector is equivalent to: $E(z,t) = E_0 \left[\hat{x} + (1+j)\hat{y} \right] e^{-j(kz-\omega t)}$

Using $(1+j) = \sqrt{2} e^{j\pi/4}$ we find, at z = 0:

$$E_x = E_0 \cos(\omega t)$$

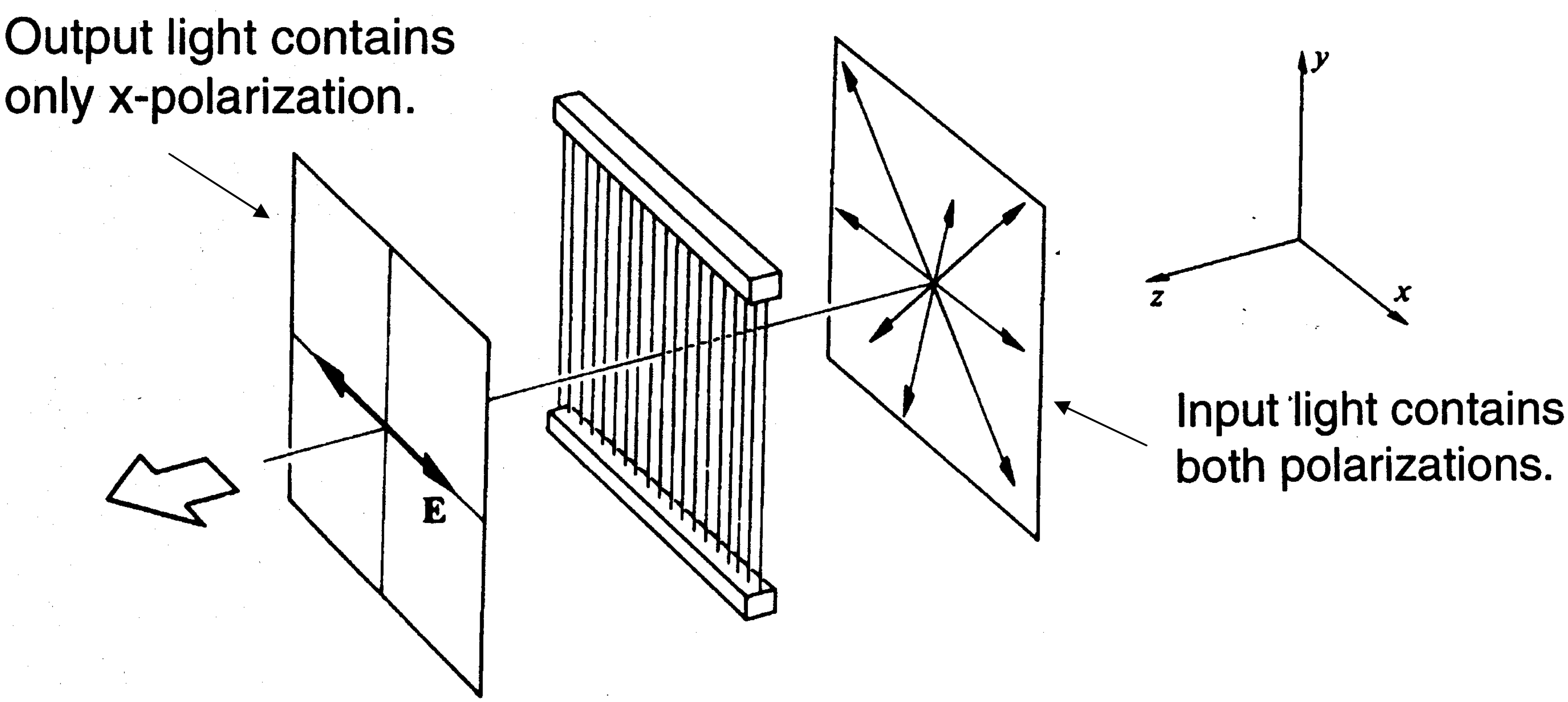
$$E_y = \sqrt{2} E_0 \cos(\omega t + \pi/4)$$

	E_x	E_y
ωt = 0	E_0	E_0
ωt = π/4	$E_0 \cdot \sqrt{2}/2$	0
ωt = π/2	0	$-E_0$
ωt = 3π/4	$-E_0 \cdot \sqrt{2}/2$	$-\sqrt{2} E_0$
ωt = π	$-E_0$	$-E_0$



left-handed elliptical polarization

A polarizer is a device which filters out one polarization component



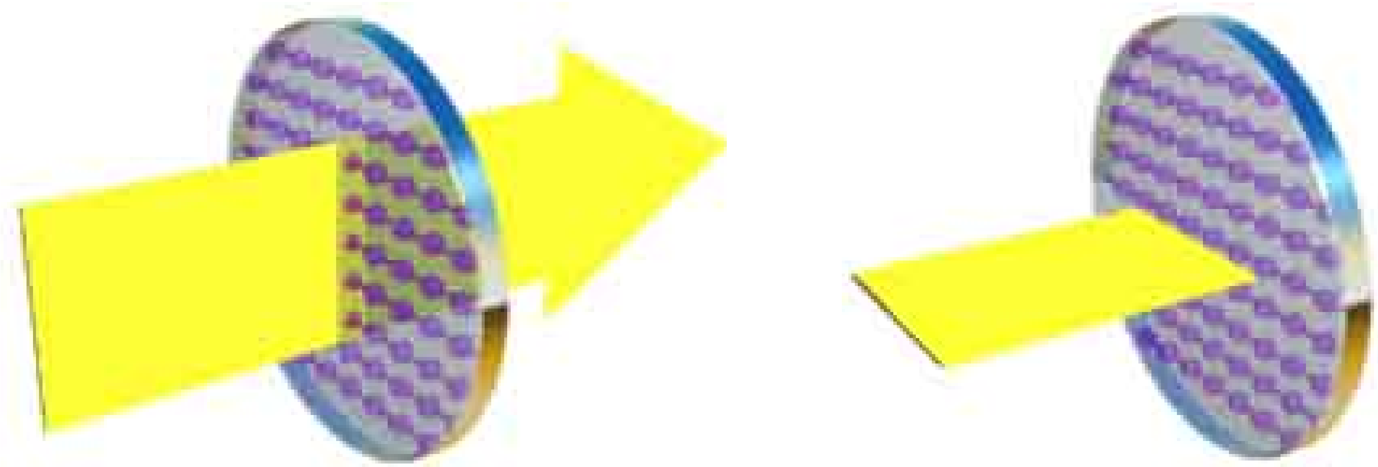
The light can excite electrons to move along the wires. These moving charges then emit light that cancels the input light. This cannot happen if the E-field is perpendicular to the wires, since the current can only flow along the wires.

Such polarizers are used most often for infrared radiation, because the wire spacing has to be much smaller than the wavelength.

Polymer-based polarizers

A polymer is a long chain molecule. Some polymers can conduct electricity (i.e., they can respond to electric fields similar to the way a wire does).

The light can excite electrons to move along the wires, just as in the case of the polymer chains.

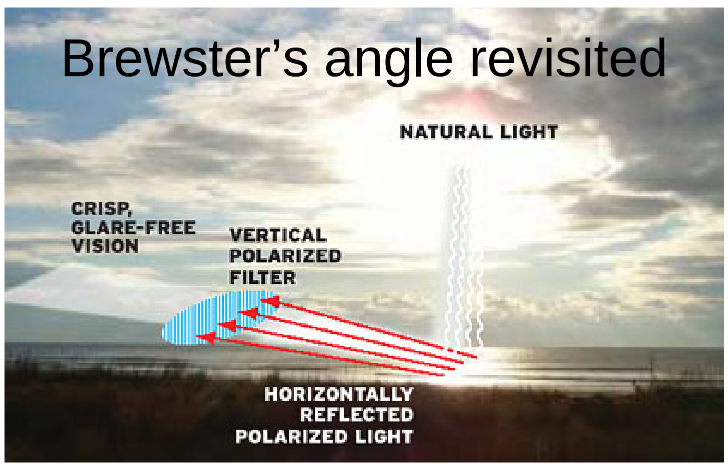


This is how polarized sunglasses work.

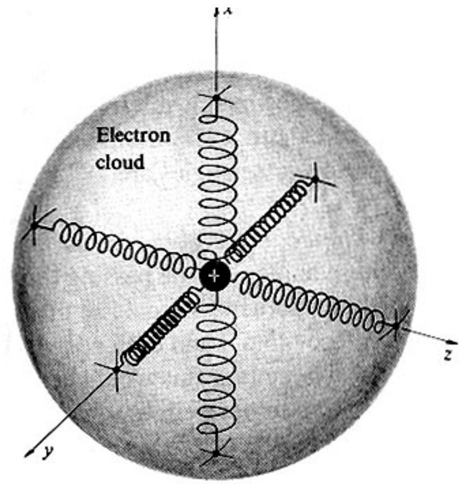
Why sunglasses are polarized

no sunglasses

sunglasses

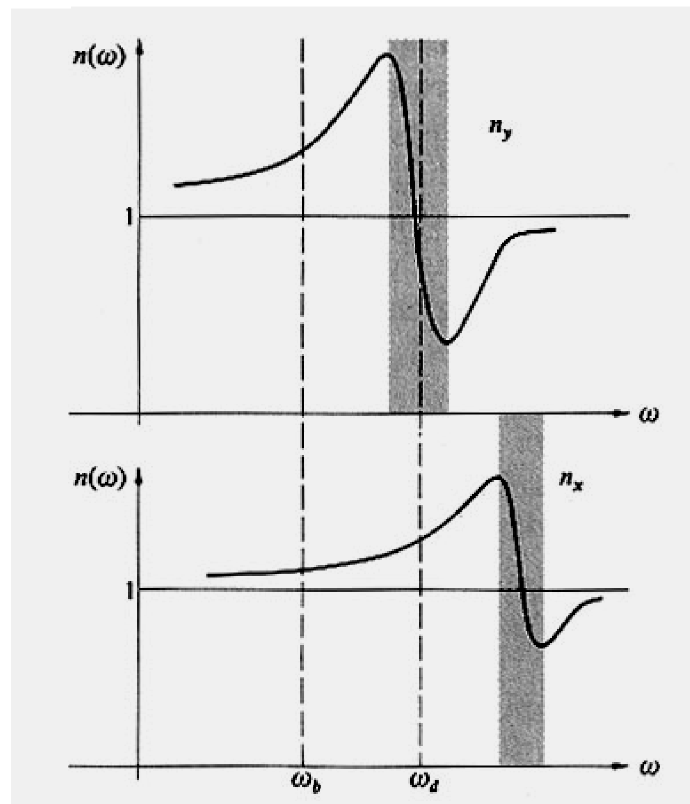


Birefringence



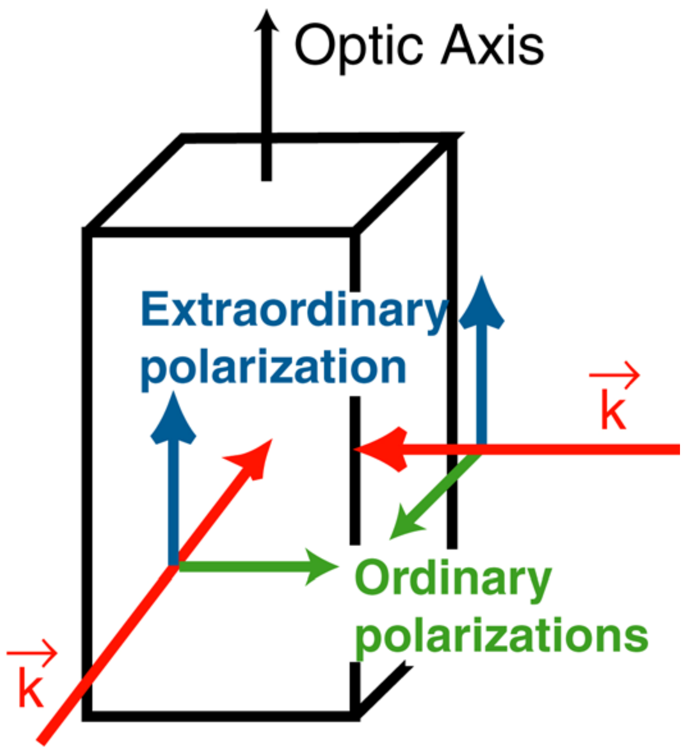
The molecular "spring constant" can be different for different directions.

The x- and y-polarizations can see different refractive index curves.



Hence, the refractive index of a material can depend on the orientation of the material relative to the polarization axis!

Uniaxial crystals have an optic axis



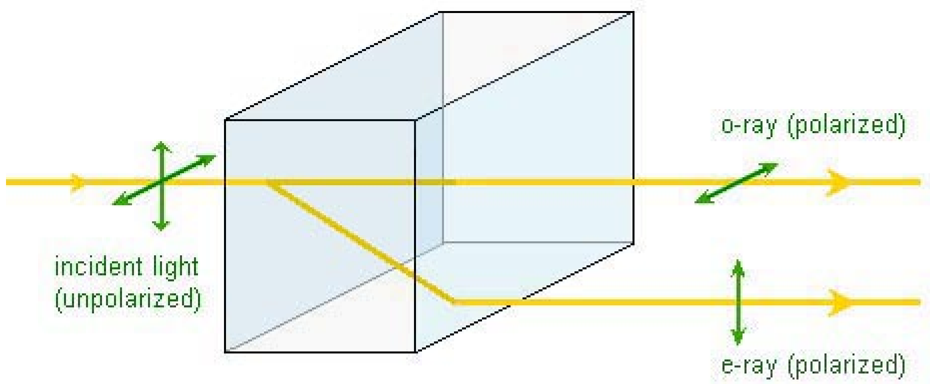
Uniaxial crystals have one refractive index for light polarized along the optic axis (n_e) and another for light polarized in either of the two directions perpendicular to it (n_o).

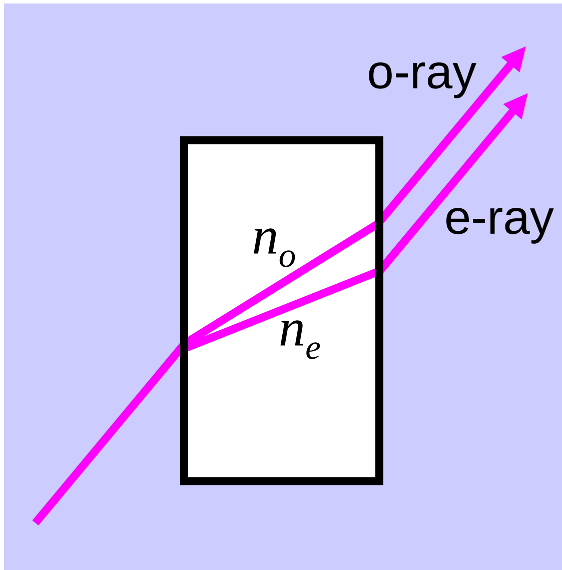
Light polarized along the optic axis is called the extraordinary ray, and light polarized perpendicular to it is called the ordinary ray. These polarization directions are the crystal "principal axes."

Light with any other polarization must be broken down into its ordinary and extraordinary components, considered individually, and recombined afterward.

Birefringence can separate the two polarizations into separate beams

Due to Snell's Law, light of different polarizations will refract by different amounts at an interface.





Birefringent Materials

Calcite, $CaCO_3$

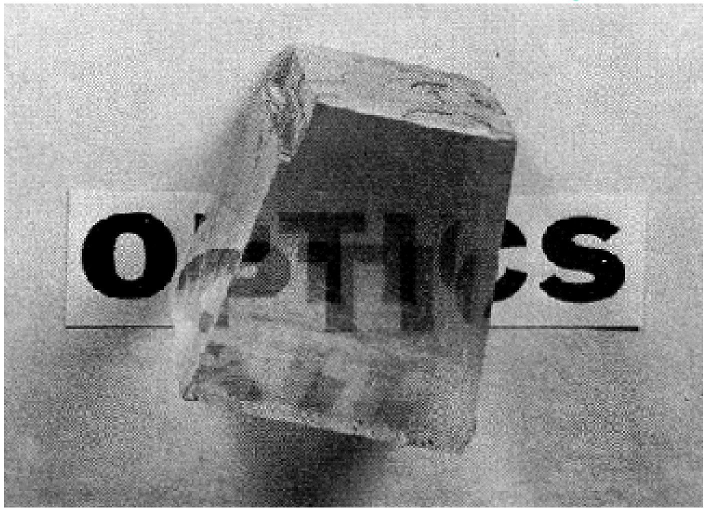
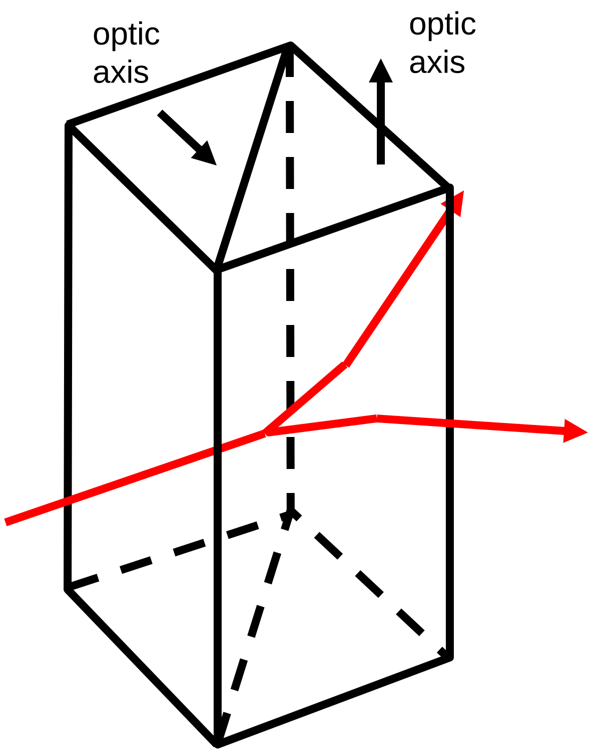


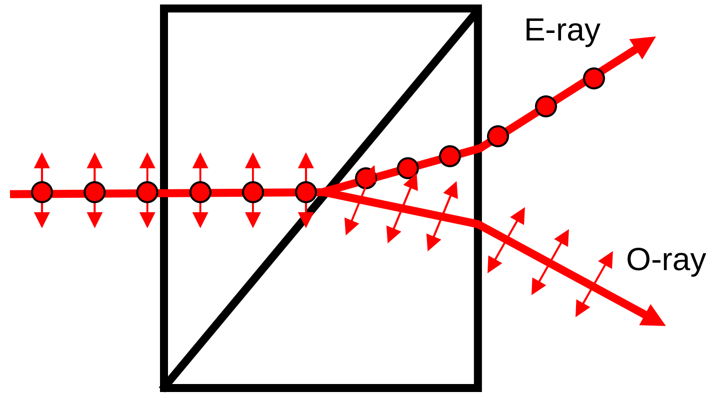
TABLE 8.1 Refractive indices of some uniaxial birefringent crystals (λ_0 = 589.3 nm).

Crystal	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1.5534
Sodium nitrate	1.5854	1.3369
Ice	1.309	1.313
Rutile (TiO_2)	2.616	2.903

Calcite is one of the most birefringent materials known.

Some polarizers use birefringence.





For example, a Wollaston prism:

Combine two calcite prisms, rotated so that the ordinary polarization in the first prism is extraordinary in the second (and vice versa).

The ordinary ray in the first prism becomes the extraordinary ray in the second one. Since $n_e < n_o$, the E-ray is refracted away from the normal to the interface. The opposite happens for the O-ray.