ELEC 262
2012 Problem set 4 solutions

1. The distance that we need is the skin depth of this conductive medium. We are considering a frequency below the plasma frequency (since the frequency of AM radio is lower than that of the plasma frequency). This is the regime where $\varepsilon/\varepsilon_0$ is a negative number, and thus the refractive index is an imaginary number. The value of *n* is:

$$n = \sqrt{1 - \frac{\omega_P^{\,2}}{\omega^2}} = \sqrt{1 - \frac{(9\text{ MHz})^2}{(0.79\text{ MHz})^2}} = 11.35\,j$$

The phase acquired by this wave as it propagates a distance z is $\exp(jnk_0z)$. But since *n* is imaginary, this is a decaying exponential. The skin depth is therefore given by $[\mathrm{Im}\{n\}\cdot k_0]^{-1}$, which is the same as $\lambda_0/(2\pi\times 11.35)$. The wavelength of the AM signal is 380 meters, so the skin depth is 5.3 meters. This is much smaller than the wavelength and also much MUCH smaller than the thickness of the ionosphere. So no AM signal leaks through at all.

2. The power radiated by an accelerating charge is given by the Larmor formula:

$$P(t) = \frac{q^2 \mathrm{a}^2}{6\pi\varepsilon_0 \mathrm{c}^3}$$

In this case, the acceleration is simply given by the centripetal acceleration of a circulating electron: $a = v^2/R$, where v is the velocity of the electron and R is the radius of the circle. Putting this (and the electron charge) into the Larmor formula, we find:

$$P(t) = \frac{e^2 c}{6\pi\varepsilon_0 \mathrm{R}^2} = 5.12\times 10^{-21}\ \text{Watts}$$

Of course, since the acceleration is pointing in the plane of the circular orbit and radially directed, there is no radiation in the radial direction. The radiation emerges tangentially to the circular orbit.

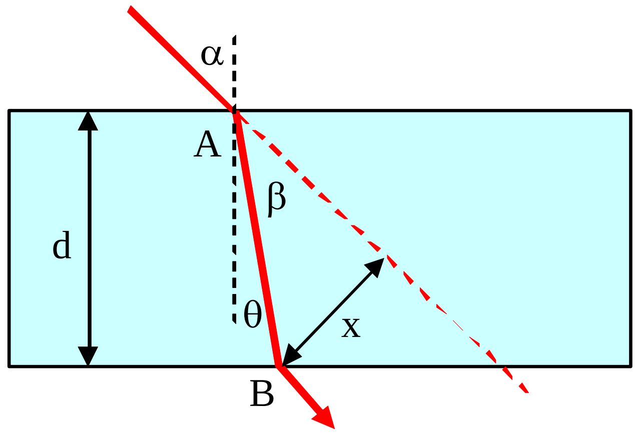
3. First we note that refraction only occurs once, at the output facet, since the angle of incidence at the input facet is 90°. We must first compute the refractive indices at the two extreme wavelengths, 770 nm and 825 nm, using the Cauchy formula for the refractive index. From Wikipedia, the values of the Cauchy coefficients for BK7 are A = 1.5046 and B = 0.0042 μm^2. We therefore find that *n*(770 nm) = 1.5117 and *n*(825 nm) = 1.5108. Now, referring to the last slide of lecture 12, we see from Snell's law that the angle of transmission (relative to the prism normal) depends on wavelength because *n* does. Since the incident angle is the same for all wavelengths, $\theta_{in} = 30°$, we find:

$\theta_{790} = \sin^{-1}(n_{790}\sin(\theta_{in})) = 49.10°$ $\qquad$ $\theta_{815} = \sin^{-1}(n_{815}\sin(\theta_{in})) = 49.06°$

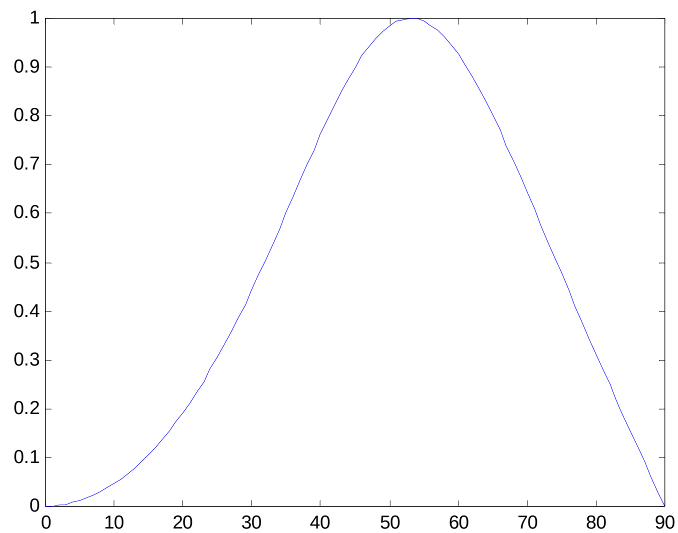
So the angular spread in the emerging beam is 0.04°. This is pretty small.

4. To solve, we refer to the drawing shown below. At the first interface, Snell's law tells us that $\sin(\alpha) = n \sin(\theta)$, which gives us the angle θ (since α and n are known). Then, the ray path AB can be found from $\cos(\theta) = d / AB$ (since the thickness d is known). Now, we observe that the angle $\beta = \alpha - \theta$. The desired detector offset is the distance x, which can be found from the angle β according to $\sin(\beta) = x / AB$. We now have two different expressions for AB – set them equal

to each other: $d / \cos(\theta) = x / \sin(\beta)$. Therefore we have our solution: $x = d \sin(\alpha - \theta) / \cos(\theta)$. For d = 1 mm, n = 1.5, and $\alpha = 30°$, we find $\theta = 19.47°$, and therefore x = 0.19 mm.



5. Unpolarized light (the incident wave) has equal amounts of parallel and perpendicular polarization. So we simply compute *R* for the perpendicular and parallel cases using the formulas given in lecture 13, slide 11, and also using $R = r^2$. We use $n_t = 1.33$, the refractive index of water. The result is shown here.



This is a plot of the degree of polarization vs. incident angle (in degrees). The degree of polarization varies between zero and unity. It is maximized at the angle where the reflected beam is 100% perpendicularly polarized – that is, Brewster's angle. For $n_t = 1.33$, this is 53.06°. However, note that the beam is substantially polarized for a very wide range of angles near Brewster's angle. In fact, we find that the degree of polarization is greater than 50% for angles θ_{in} between 32° and 74°, a very wide range. That's why it is easy to see polarization effects using your sunglasses and any body of water on a sunny day.