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Broadband group-velocity anomaly in transmission through a terahertz photonic crystal slab

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We have measured the complex transmission coefficient of a two-dimensional photonic crystal slab for propagation perpendicular to the slab, using terahertz time-domain spectroscopy. At high frequencies, where the lattice parameter exceeds the wavelength, we find that the group delay is equal to that of empty space despite the fact that the volume-weighted average dielectric of the slab is $\varepsilon_{\text{ave}} \sim 3.8$. In contrast to most other examples of anomalous group delay effects, this unusual phase transparency persists over a very broad spectral bandwidth, demonstrating that such anomalies do not require the presence of a nearby isolated resonance. Our results are consistent with finite-difference time-domain simulations.

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Observations of unusual behavior in the group velocity of propagating light waves are of considerable interest. The possibility of a superluminal group velocity near a resonance has been recognized for many years.[1] Numerous approaches have been described for observing pulse propagation in a regime of superluminal or negative group velocity, including operation near a resonant gain line,[2,3] penetration through a tunnel barrier,[4] and propagation in the transparent regime between two closely spaced absorption resonances.[5] Similar effects can be observed in photonic crystal devices,[6,7] as well as in the propagation of electronic signals.[8] Such effects generally rely on the rapid variation of the refractive index with frequency which occurs near a resonance or when the wavelength is close to a characteristic length scale of the medium. As a result, the regime of anomalous group velocity is confined to a fairly narrow portion of the spectrum.

Here we report an observation of a *broadband* group velocity anomaly in the transmission of a picosecond terahertz pulse through a photonic crystal slab. For wavelengths shorter than the lattice parameter of the periodic structure, we find an abrupt transition to a regime of zero phase dispersion. This anomaly persists for at least hundreds of GHz, occupying a fractional bandwidth greater than 50%. In this spectral range the group velocity v_G is equal to the vacuum speed of light, despite the fact that the volume-weighted average dielectric constant of the sample is $\varepsilon_{\text{ave}} \sim 3.8$.

We consider propagation of radiation through a photonic crystal slab, in a direction perpendicular to the plane of the slab, rather than the more familiar case of in-plane propagation.[9,10] Recently, out-of-plane propagation has attracted a great deal of attention in the photonic crystal community, due to the significance of out-of-plane loss mechanisms.[11–15] For propagation perpendicular to the slab, calculations of the long-wave (homogeneous) limit pose special challenges.[16] In addition, guided resonances, which couple to freely propagating out-of-plane modes, have recently been the subject of intense study.[17–21] Despite the significance of out-of-plane propagation, there have been only a few experimental reports of transmission or reflection spectra,[11,12,15,21–23] and none with sensitivity to spectral phase.

Our experimental technique is based on terahertz time-domain spectroscopy (THz-TDS).[24] We employ THz-TDS to determine the complex transmission coefficient of a photonic crystal over a broad spectral bandwidth (see Fig. 1).[9,10,25–28] The sample is a slab (305 μm thick) of high-resistivity ($\rho > 10^4\ \Omega$ cm) silicon (with refractive index $n = 3.42$), with an array of circular holes etched all the way through, using deep reactive ion etching. The holes have diameters of 360 μm, and are arranged on a hexagonal lattice with a pitch of $a = 400\ \mu$m. The remaining dielectric occupies ~26% of the slab, so the volume-weighted average dielectric is $\varepsilon \sim 3.8$. In order to confirm that the bandwidth of the incident pulse is broad enough to extract useful spectroscopic information up to 1.5 THz, we also measure a silicon slab of equal thickness but with no holes. In order to avoid collecting radiation that is coherently diffracted from the sample, we use a collection lens which subtends a fairly small solid angle, such that only the directly transmitted portion of the terahertz beam is detected.

In a coherent spectroscopic technique such as terahertz time-domain spectroscopy (THz-TDS), one measures the real electric field of the wave transmitted through a sample $E_S(t)$, and compares to a reference measurement $E_R(t)$, typically one in which the sample has been removed from the beam.[24] This permits a direct measurement of the phase difference $\Delta\phi = \phi_S - \phi_R$, which can be related to an effective refractive index according to $\Delta\phi = (n_{eff}(\omega) - 1)\omega L/c$. Here, L is the slab thickness and c is the light speed in vacuum. Although we do not require the effective index $n_{eff}(\omega)$ for

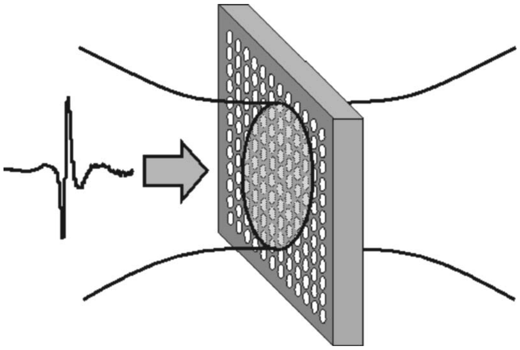

FIG. 1. A schematic of the experimental arrangement. The photonic crystal slab is illuminated at normal incidence to the plane of the periodicity. The THz beam spot size is large enough to illuminate many holes.

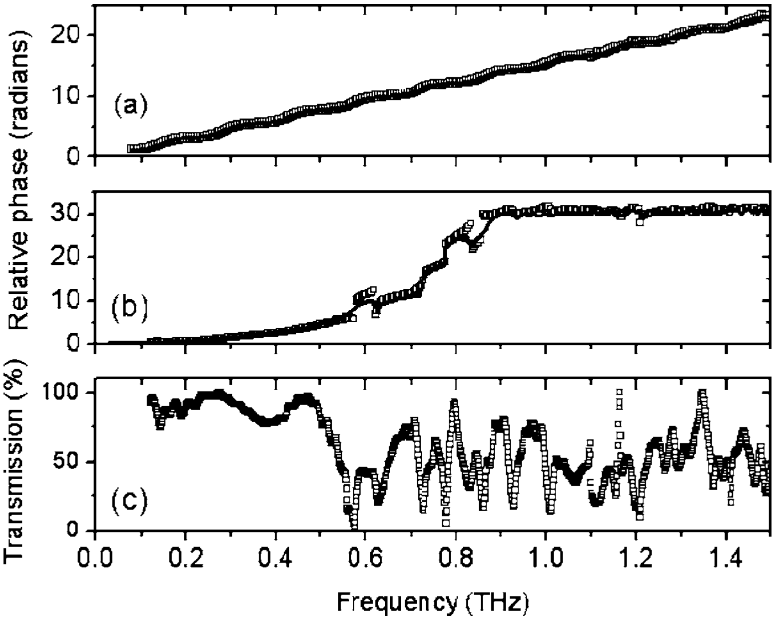

FIG. 2. The measured relative phase, $\Delta\phi = \phi_S - \phi_R$, for (a) a solid silicon slab and (b) a photonic crystal slab. In (a), the solid curve is a calculation of the Fabry-Pérot phase, using the known thickness and frequency-independent refractive index. This shows that the measured results are reliable up to at least 1.5 THz. In (b) the solid curve is a finite-difference time-domain (FDTD) simulation. Above 0.9 THz, the relative phase is nearly constant, indicating a group velocity equal to the vacuum speed of light in this broad spectral range. (c) The amplitude transmission spectrum, $|E_S(\omega)/E_R(\omega)|$, for the photonic crystal slab. The spectrum shows no obvious signatures corresponding to the abrupt change in the phase spectrum at 0.9 THz.

subsequent analysis, it is helpful to extract this quantity in order to explore the role of multiple reflections inside the slab. Such Fabry-Pérot effects can distort the extracted parameters, so care must be taken in the numerical analysis. In particular, such effects can be difficult to disentangle in the case where the thickness of the sample is comparable to or less than the coherence length of the pulse. Here, we obtain the effective refractive index using a numerical error minimization procedure described previously.[29] In our case, this procedure leads to only a relatively minor correction to the result.

We have recently described the first measurements of the dispersion in photonic crystal slabs for the case of propagation perpendicular to the plane of the slab.[30] At relatively low frequencies, we observed a strong variation in the phase superimposed on phase jumps associated with the well-known guided resonances.[17,18] However, as shown in Fig. 2, at higher frequencies there is a fairly abrupt transition at ~0.9 THz to a nearly frequency-independent plateau. This region of constant $\Delta\phi$ persists over a wide spectral range, extending beyond 1.5 THz, the high-frequency limit of our measurement. One can compute the group velocity directly from the measured phase difference

$$v_G(\omega)/c = \left[1 + \frac{c}{L} \cdot \frac{d}{d\omega}(\Delta\phi)\right]^{-1}.$$

Evidently, if $\Delta\phi$ is independent of ω (as we observe experimentally), then the group velocity is equal to c. That is, the group delay for propagation through the slab, $\tau_G = d\phi/d\omega$, is given by L/c, equivalent to the group delay for propagation through a distance L of vacuum. The pulse envelope of an optical pulse propagating in this medium traverses the slab as rapidly as if the slab was removed from the optical beam.

This surprising effect is even more dramatic when viewed in the time domain. We first apply a frequency-domain window to the measured THz pulses in order to remove all spectral content below 0.9 THz, where the propagation is strongly dispersive. We then apply an inverse Fourier transform, and show the resulting filtered wave forms in the time domain (see Fig. 3). Of course, the spectral windowing introduces distortions in the wave forms, but since the same window is used for all three cases it is reasonable to make comparisons among them. In these time-domain data, it is clear that the group delay for transmission through the photonic crystal is the same as for air, and quite distinct from that of a solid silicon slab of equal thickness. In essence, the radiation above 0.9 THz acts as if it propagates only through the holes and not through the solid dielectric, despite the fact that the dielectric is transparent and occupies about one fourth of the illuminated area.

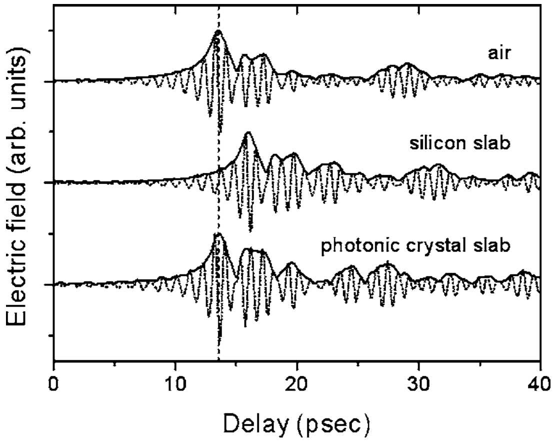

FIG. 3. Time-domain wave forms after propagating through air, a solid silicon slab, and the photonic crystal slab. These have been filtered to remove all spectral content below 0.9 THz. The solid curves show the pulse envelopes, while the dashed curves show the real electric field. The group delay (transit time) through the photonic crystal slab is identical to that of air (vertical dashed line), whereas the middle (solid silicon) wave form is delayed by $\Delta t = (n_{Si} - 1)L/c$ relative to the other two. Curves vertically offset for clarity.

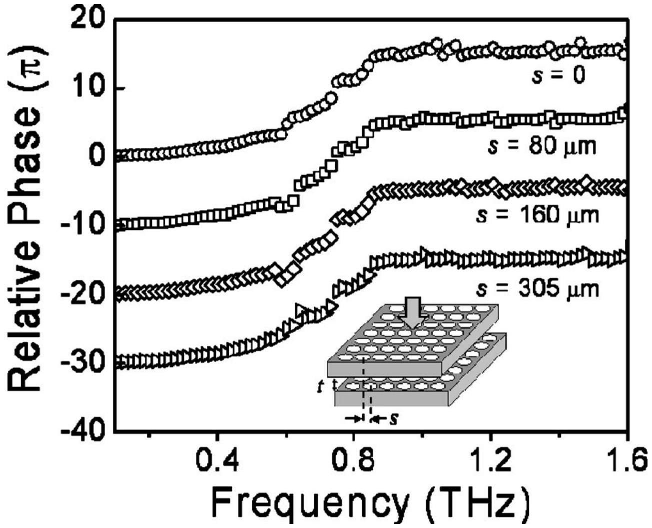

FIG. 4. The measured relative phase, $\Delta\phi=\phi_S-\phi_R$, for the two-slab structure shown in the schematic inset for fixed spacing t $=80$ μm and different lateral shifts s. The relative phase is nearly constant above 0.9 THz regardless of the value of s, indicating the same broadband group velocity anomaly as observed in a single slab. This demonstrates that the radiation cannot simply be passing through the holes, since the solid portions of one of the slabs must partially cover the holes of the other one at certain values of s. Curves vertically offset for clarity.

Our results bear some similarities to recent studies of macroporous silicon filters.[31,32] However, in those cases the solid dielectric is a strong absorber, so that the transmitted light must pass entirely through the pores and not through the dielectric. In this situation, one can describe the resulting spectra by appealing to the waveguiding effects of the pores. However, we can explicitly demonstrate that this is not the case in our systems, by using two identical slabs arranged in a sandwich structure. Figure 4 shows the relative phase $\Delta\phi$ for a two-slab structure,[19,33] with a fixed spacing $t=80$ μm and several different values of the lateral shift s of one slab relative to the other. As the lateral shift varies, the solid portion of one slab shadows the air holes of the other one, with the degree of occlusion varying from zero (for $s=0$) to a maximum of 33.8% (for $s=200$ μm). The group anomaly is qualitatively unchanged in all cases, even those with the maximum degree of occlusion. Also, through the same filtering and inverse Fourier transform procedures described above, we can obtain equivalent time-domain results as shown in Fig. 3. These results eliminate the possibility of a simple waveguide description for the observed phenomenon.

This phase transparency is related to the finite thickness of the photonic crystal slab. To demonstrate this, we examine the out-of-plane band structure of a photonic crystal slab of infinite thickness. Figure 5 shows this band structure calculated with a plane-wave expansion method.[34] The band increases nonlinearly with the out-of-plane wave vector, in contrast to our experimental observations of a frequency-independent $\Delta\phi$ (which would require a linear band). The effective index calculated from this band structure is close to that of bulk silicon at frequencies above 0.9 THz, inconsistent with the observed group velocity in this spectral range.

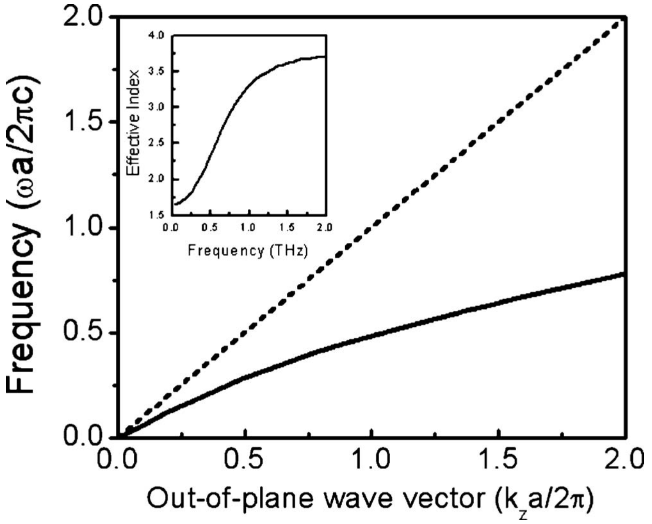

FIG. 5. The out-of-plane band structure of a photonic crystal slab of infinite thickness, calculated with the plane-wave-expansion method. The dashed line is the light line, $\omega=c\mathbf{k}_z$. The band (solid line) increases nonlinearly with frequency, a result which is inconsistent with the observed behavior in the range above 0.9 THz. The inset shows the effective index computed from this band structure.

Our results suggest the existence of a unique feature in the out-of-plane band structure of finite-thickness photonic crystal slabs. We would expect that at least one strongly coupled band lies parallel to the light line in a broad spectral range. Unfortunately, for normal incidence ($\mathbf{k}_{\parallel}=0$), such band structures are very difficult to compute for a finite-thickness slab, because the wave experiences no periodicity along the propagation direction. Furthermore, the surfaces of constant phase inside the material are not planar, but rather are corrugated with the periodicity of the lattice.[16] Finally, accurately accounting for the finite thickness of the slab poses serious computational challenges.[35] So, here we rely instead on finite-difference time-domain (FDTD) calculations to simulate our results. The solid curve in Fig. 2(b) shows a FDTD simulation which accurately reproduces all of the significant features of the data, in particular the abrupt transition to zero dispersion at 0.9 THz.

In conclusion, we report what is to our knowledge the first example of an anomalous group delay which extends over a broad fractional bandwidth, $\Delta\omega/\omega>50\%$. This striking behavior is observable only in the phase of the transmitted light, with no obvious signatures in the amplitude spectrum. The broad bandwidth indicates that the effect is not related to the dispersion in the vicinity of a single resonance, but rather is a consequence of the characteristics of the out-of-plane photonic band structure. Further work will be required to understand if this is a general feature of high contrast photonic crystal slabs, or if it is specific to a particular set of structural parameters. This unusual result could provide a new method for manipulating light waves over a broad spectral bandwidth.

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